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WAVEAMP:
A Program for Computation of Wave Elevations Created by a Ship Travelling at a Constant Speed

Fotis A. Papoulias and Robert F. Beck
Department of Naval Architecture and Marine Engineering

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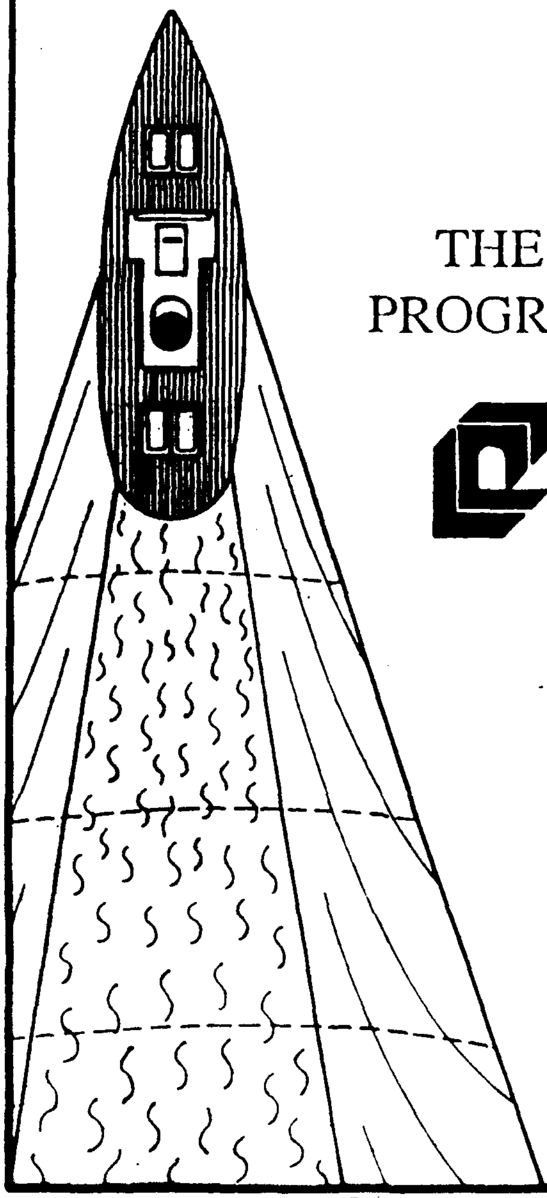

THE UNIVERSITY OF MICHIGAN
PROGRAM IN SHIP HYDRODYNAMICS

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ABSTRACT

This report documents the computer code WAVEAMP, a program to compute the Kelvin wave amplitudes using the Nuemann-Kelvin approximation. To run WAVEAMP the source strength on the hull surface must first be determined using a Neumann-Kelvin code.

Example output is presented for the Quapaw hull at three Froude numbers. The calculations include both near-field and far-field waves, and longitudinal wave cuts.

ACKNOWLEDGEMENTS

This work was initiated under Contract No. N000167-86-K-0139 from the David Taylor Research Center. Partial support was also received from the Program in Ship Hydrodynamics at the University of Michigan, funded by the University Research Initiative of the Office of Naval Research, Contract No. N000184-86-K-0684.

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INTRODUCTION AND OUTLINE

This report is divided into four sections. A brief theoretical background and problem formulation is included in Section I. The numerical method used is described in Section II. The method results in a considerable reduction in computer time at the expense of slight loss of accuracy. In Section III results of predicted wave amplitude for the QUAPAW hull are presented. Finally, user's instructions for program WAVEAMP are described in Section IV. A program listing is included in the Appendix.

I. THEORETICAL BACKGROUND

In this section, a brief outline of the theoretical background needed for the computation of ship waves is presented. Details of the material included here can be found in [1] and [2].

1.1 Problem Formulation

In the following, a right handed coordinate system is used with the origin at the intersection of the calm water plane and midship. The positive x-axis points out the bow and the z-axis in position upward.

The total potential for a thick ship advancing with a constant speed V in an otherwise calm, inviscid and incompressible fluid is given by

$$\Phi = - Vx + \phi \quad , \tag{1}$$

where ϕ is the perturbation potential which satisfies Laplace's equation

$$\nabla^2 \phi = 0 \tag{2}$$

The linearized free-surface boundary condition is

$$\phi_{xx} + k_o \phi_z = 0 \quad \text{on} \quad z = 0 \quad , \tag{3}$$

where $k_o = g/V^2$. A fundamental solution of (2) and (3) is the Green function, given by

$$G(P,Q) = \frac{1}{r} - \frac{1}{r'} + \frac{k_o}{\pi} \lim_{\mu \to 0} \int_{-\pi}^{\pi} d\theta \int_0^{\infty} dk \frac{\exp\{k[z+\zeta+i(x-\xi)\cos\theta+i(y-\eta)\sin\theta]\}}{k_o - k\cos^2\theta - i\mu\cos\theta} \tag{4}$$

where $P \equiv (x,y,z)$ is the field point, $Q \equiv (\xi,\eta,\zeta)$ is a point source of strength -4π , and

$$r, r' = [(x-\xi)^2 + (y-\eta)^2 + (z \mp \zeta)^2]^{1/2} \quad . \tag{5}$$

In the usual method of potential theory, Green's theorem can be applied to a large volume of fluid containing the body, and extending to infinity both laterally and in depth. The following result for the perturbation potential in terms of a pure source density distribution σ is obtained:

$$\phi(P) = -\frac{1}{4\pi}\iint_{S_H} G(P,Q)\sigma(Q)dS(Q) - \frac{1}{4\pi k_o}\int_{C_H} \nu_x\, G(P,Q)\sigma(Q)d\eta \quad , \tag{6}$$

where ν_x is the x-component of the normal vector out of the fluid into the source domain, S_H is the wetted surface of the body, and C_H is the intersection of the hull and the calm water free surface.

An integral equation for the source strength may be found by differentiating (6) with respect to the normal on the body and setting it equal to the body boundary condition which requires that the normal velocity on the hull be zero. Using (1) and (6), we may write

$$Vn_x = -\frac{1}{2}\sigma - \frac{1}{4\pi}\iint_{S_H} G_n(P,Q)\sigma(Q)dS(Q) - \frac{1}{4\pi k_o}\int_{C_H} \nu_x G_n(P,Q)\sigma(Q)d\eta \tag{7}$$

Solution of (7) is achieved by discretizing the surface of the body into plane quadrilateral panels using the method of Hess and Smith, as described in [1] and [2]. The equations are then assembled to yield a system of linear algebraic equations in terms of the unknown source strength σ . The source strengths and the body configuration can then be used as input for the wave amplitude computations.

I.2 Determination of Wave Amplitudes

The linear free surface elevation ζ at any point (x,y) is given by

$$\zeta = \frac{V}{g}\,\phi_x \Big|_{z=0} \tag{8}$$

where the potential ϕ us given by (6). In order to compute ϕ_x we need the x-component of the gradient of the Green function (4), which may be decomposed as

$$G_x = G_x^{(R)} + G_x^{(R')} + G_x^{(P)} + G_x^{(L)} \quad . \tag{9}$$

The first two terms in (9) reflect the influence of the $1/r$ and $1/r'$ terms (the Rankine terms) in (4), and cancel out on the $z = 0$ plane. Therefore,

we need to evaluate only the terms due to the double integral over the body surface S_H $(G^{(P)})$ and the terms due to the single integral along the waterline $C_H(G^{(L)})$. By representing each panel by a single concentrated source at its centroid (x_c, y_c, z_c) we may find

$$G_x^{(P)} = \frac{2k_o^2 A}{\pi} \int_{-\pi/2}^{\pi/2} \sec^3\theta \, \mathrm{Im}\{F_o[k_o \sec^2\theta (Z_F + Z_C)]\} d\theta \quad , \tag{10}$$

$$G_x^{(L)} = \frac{2k_o^2 \nu_x \Delta y_L}{\pi} \int_{-\pi/2}^{\pi/2} \sec^3\theta \, \mathrm{Im}\{F_o[k_o \sec^2\theta (Z_F + Z_L)]\} d\theta \quad . \tag{11}$$

$Fo(z)$ is given by

$$F_o(z) = e^z \{E_1(z) - 2\pi i \, H[\mathrm{Im}(-z)]\} - \frac{1}{z} \quad , \tag{12}$$

where H is the Heaviside step function and $E_1(z)$ the complex exponential integral

$$E_1(z) = \int_z^{\infty} \frac{e^{-t}}{t} dt \quad . \tag{13}$$

In equations (10) and (11), A is the area of the source panel, and

$$Z_F = \quad i(x_F \cos\theta + y_F \sin\theta) \quad , \tag{14}$$

$$Z_C = z_c - i(x_c \cos\theta + y_c \sin\theta) \quad , \tag{15}$$

$$Z_L = \quad -i(x_L \cos\theta + y_L \sin\theta) \quad , \tag{16}$$

$(x_F, y_F, 0)$ = the chosen field point on $z = 0$

(x_c, y_c, z_c) = the centroid of the source panel

$(x_L, y_L, 0)$ = the centroid of the line segment of the source panel that coincides with the free surface

Δy_L = the transverse component of this line segment

ν_x = the x-component of the unit normal of the segment.

The sign correction for Δy_L is derived from the fact that the line integral in (7) is performed in a counter-clockwise sense.

II. NUMERICAL CONSIDERATIONS

The majority of effort in the numerical evaluation of the wave amplitudes is spent in the computation of the θ-integrals in equations (10) and (11). To facilitate the computation we denote the θ-integral by

$$I = -\int_{-\pi/2}^{\pi/2} \sec^3\theta \, \mathrm{Im}\left\{F_0\{[z+i(x\cos\theta + y\sin\theta)]\sec^2\theta\}\right\}d\theta \quad , \tag{17}$$

where (x,y,z) is the dimensionless (non-dimensionalized by k_0) distance between source and field point. The main difficulty in evaluating (17) is due to the behavior of the $F_0(\theta)$ function versus θ. Typical graphs of $F_0(\theta)$ are shown in Figures 1 through 6, and the following comments can be made:

(i) $F_0(\theta)$ is a rapidly oscillating function in θ when z is close to zero (i.e., the source point approaches the free surface), or when x and y are (in absolute value) large compared to z. This shows that it is more difficult to evaluate the far-field wave pattern and especially the waves generated by the panels located near to the free surface, which give the largest contribution to the radiated waves. This fact renders wave-amplitude computations more sensitive to the θ-integration (17) than the wave-resistance (Neuman-Kelvin) problem.

(ii) $F_0(\theta)$ is an even function in θ, i.e., $F_0(-\theta) = F_0(\theta)$, when $y = 0$. Even off the centerline, $y = 0$, the graph of $F_0(\theta)$ vs. θ is almost symmetric with respect to $\theta = 0$ when $|x| >> |y|$, i.e., far downstream.

(iii) $F_0(\theta)$ is an odd function of y, which means that the contribution from the line integral (11) is zero right on the ship's centerline. This fact is in agreement with symmetry considerations. Off the centerline, the line integral

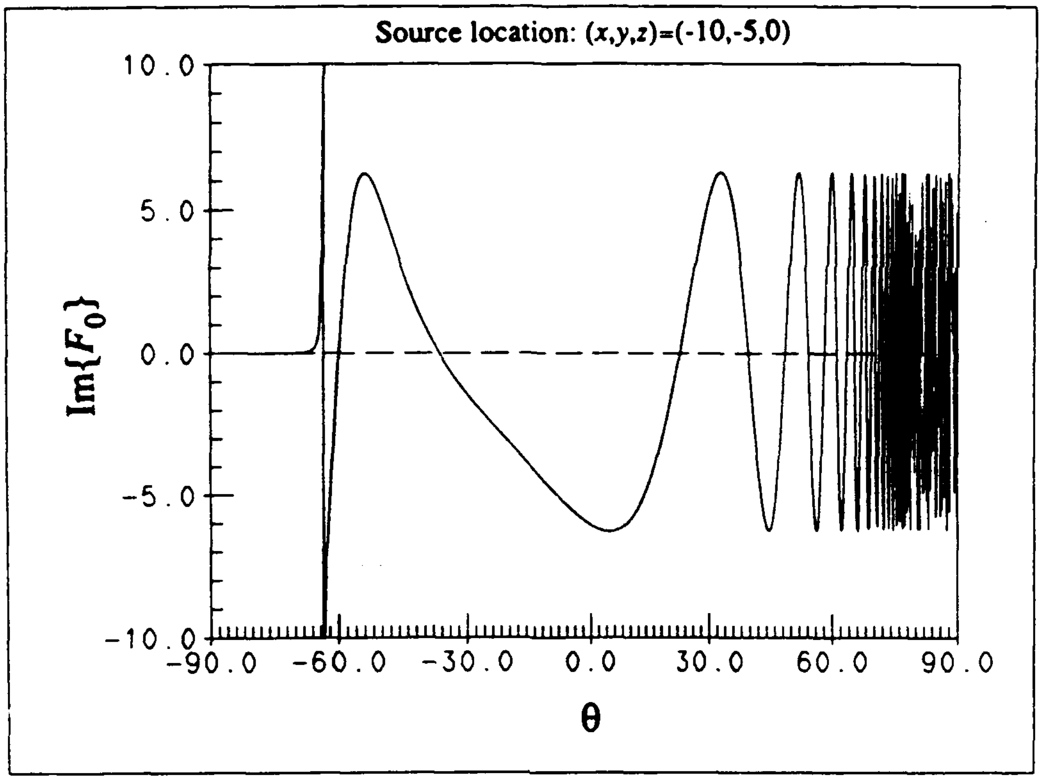

Figure 1. Im{$F_o(\theta)$} for (x,y,z) = (-10, -5, 0)

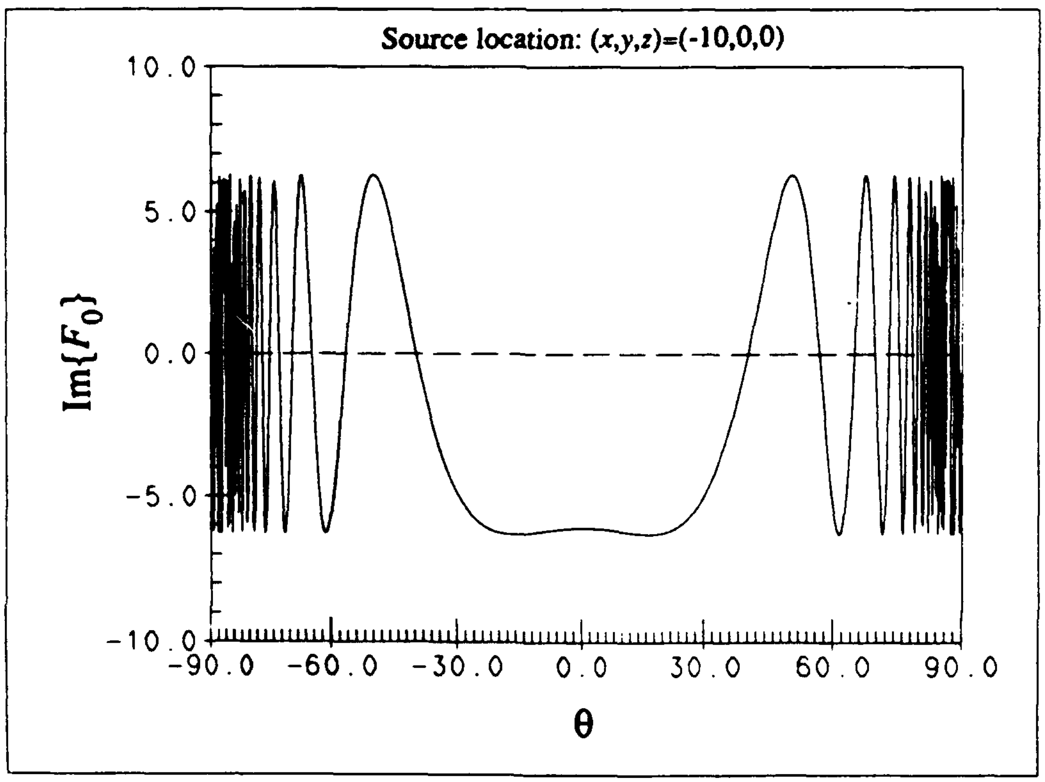

Figure 2. Im{$F_o(\theta)$} for (x,y,z) = (-10, 0, 0)

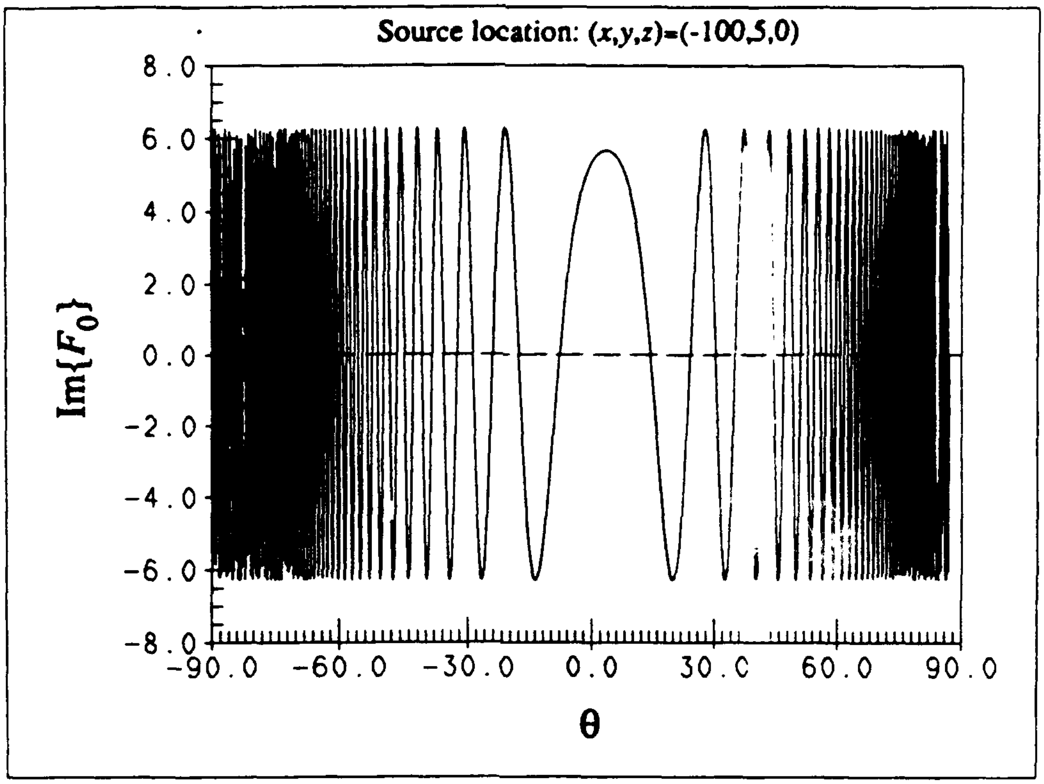

Figure 3. $\mathrm{Im}\{F_o(\theta)\}$ for $(x,y,z) = (-100, 5, 0)$

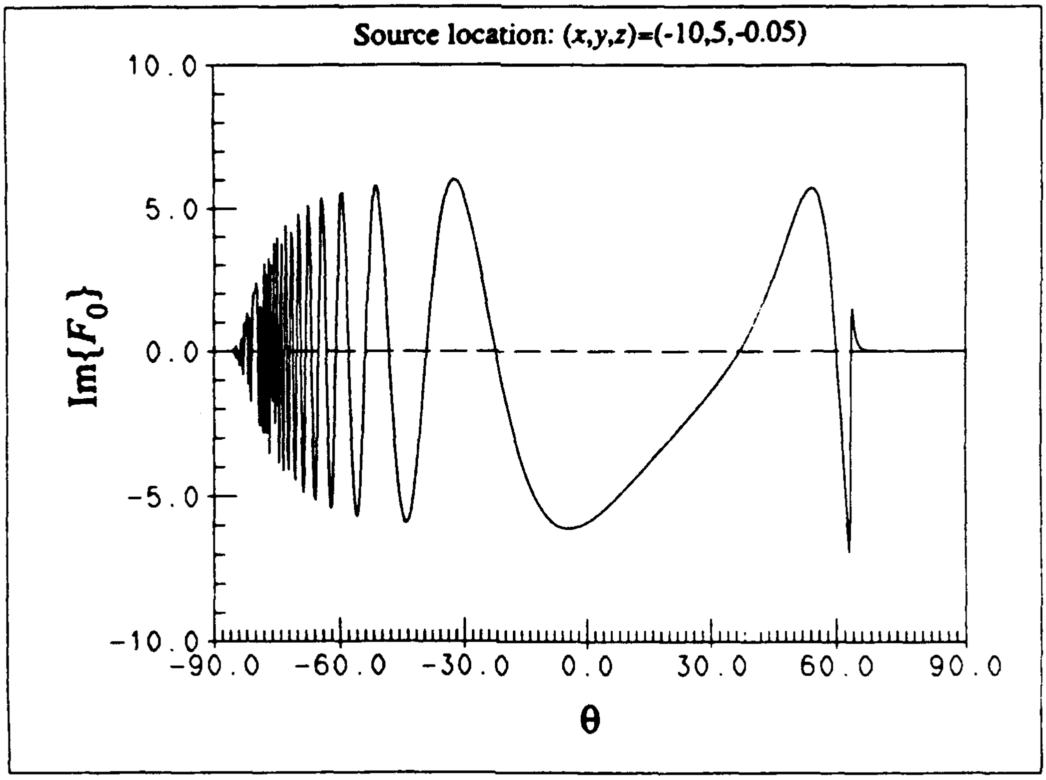

Figure 4. $\mathrm{Im}\{F_o(\theta)\}$ for $(x,y,z) = (-10, 5, -0.05)$

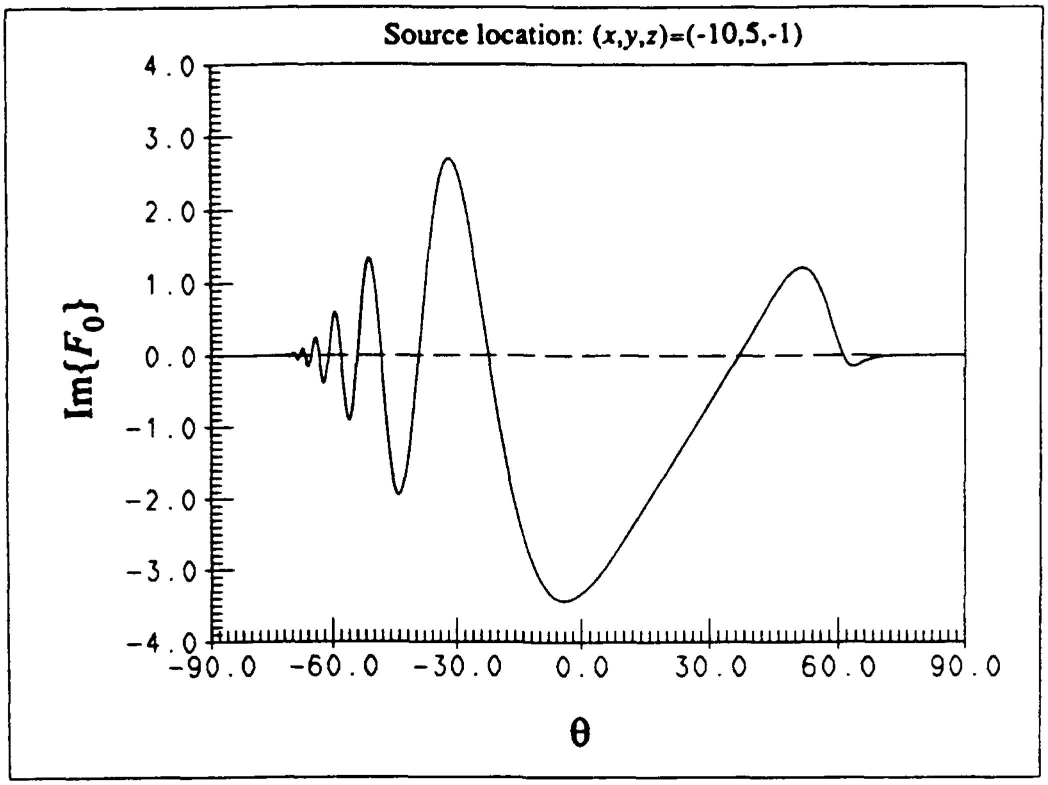

Figure 5. Im{$F_0(\theta)$} for (x,y,z) = (-10, 5, -1)

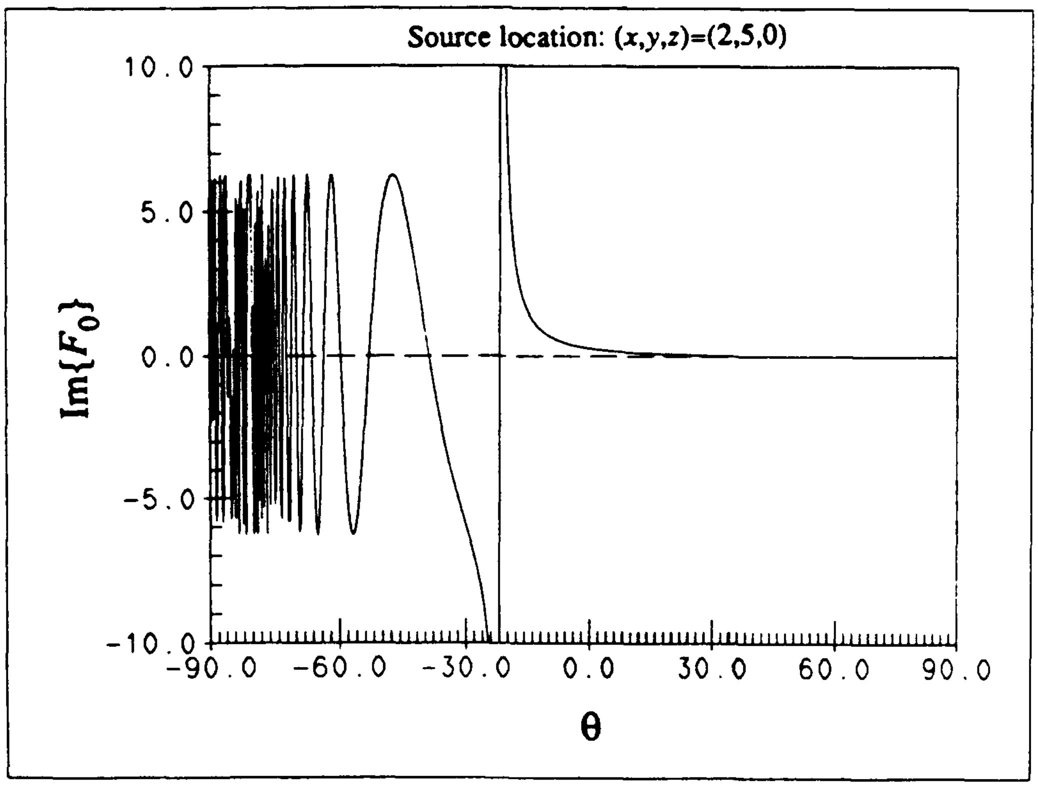

Figure 6. Im{$F_0(\theta)$} for (x,y,z) = (2, 5, 0)

contribution approaches zero as the field point moves far downstream.

The integrand (17) possesses two integrable singularities at $\theta = \pm \pi/2$ and another integrable one when $z = 0$ at $\theta = -\tan^{-1}(x/y)$ due to the logarithmic-type singularity of the complex exponential integral $E_1(0)$. In the following numerical computations, the distance between source and field point is changed from Cartesian (x,y,z) to cylindrical (R,ψ,Z) as shown in Figure 7. In addition, the limits of integration in (17) are taken to be $-\pi/2 + \varepsilon$ and $\pi/2 - \varepsilon$, where ε is a small cut-off number. Values of integral (17) versus the number of points in the θ-integration, N_θ, are shown in Figures 8 and 9 for different values of ε and source-field point distance. From these figures it can be seen that: (a) integral (17) is well behaved in the limit of $\varepsilon \to 0$ and the result is insensitive to the actual value of ε; and (b) when the field point is located off the x-axis the convergence of (17) is much slower.

Based on the above remarks we can compute integral (17) for a range of (x,y,z) values expressed in the cylindrical coordinate system (R,ϕ,z) of Figure 7. The computations have been performed for the following range of (R,ϕ,z) values:

R ranges from 1 to a 100 with increments of 1.

ϕ ranges from 90° to -60° with increments of 10°, and from -60° to -90° (inside the Kelvin cusp line) with increments of 0.5°.

z takes the values -5, -2, -1.5, and from -1.0 to -0.1 with increments of -0.1.

The resulting values of integral (17) (called "influence coefficients") have been tabulated and can be used to evaluate the influence of a source panel on any field point within the above range of (R,ϕ,z) values. Typical

variation of the influence coefficients versus z is shown in Figures 10, 11, and 12. Values at $z = 0$ have been estimated using parabolic extrapolation. Rapid decay of the influence coefficients versus R outside of the Kelvin angle is demonstrated in Figure 13, whereas their persistence vs. R for ϕ in the vicinity of the Kelvin cusp ($\approx -70.72°$) is shown in Figures 14 and 15. The same trend is evident from the graphs versus ϕ in Figures 16, 17, and 18. Finally, the influence of any source point on the particular field point is estimated by using the above tabulated results and trilinear interpolation in the (R,ϕ,z) coordinates. This technique is only an approximate one since interpolation errors are inevitable. It provides, however, an efficient and low cost computation of a ship's Kelvin wake.

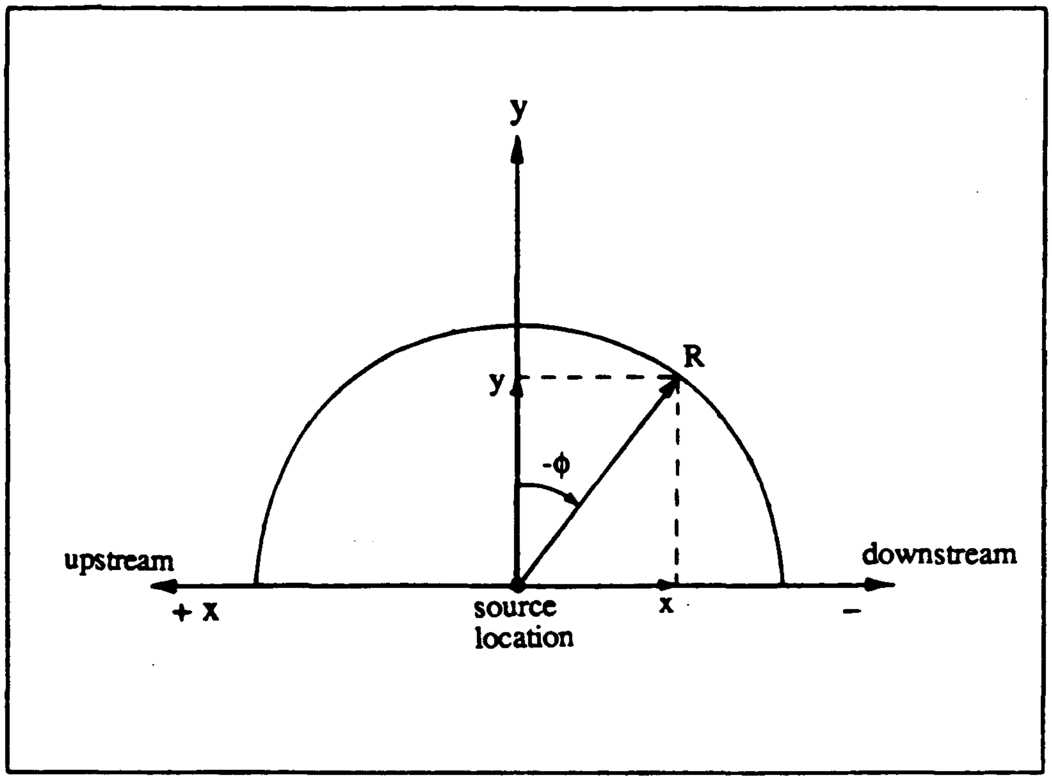

Figure 7. Coordinate transformation

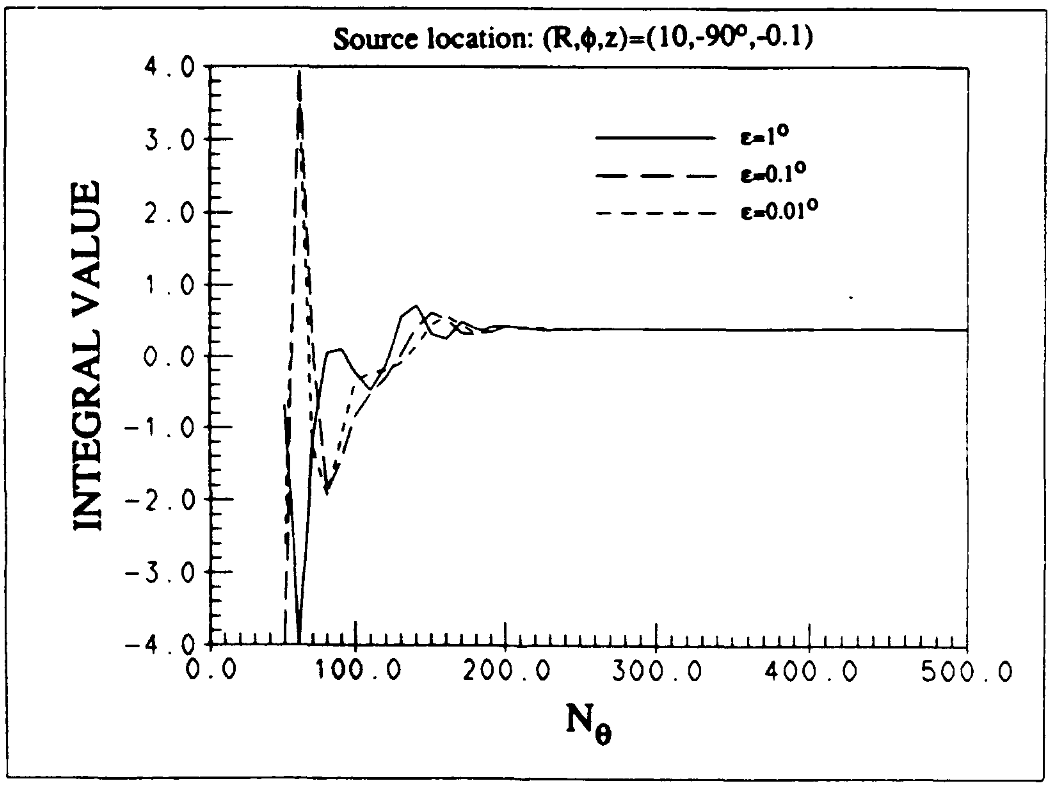

Figure 8. Convergence of integral (17) for $(R,\phi,z) = (10, -90°, -0.1)$

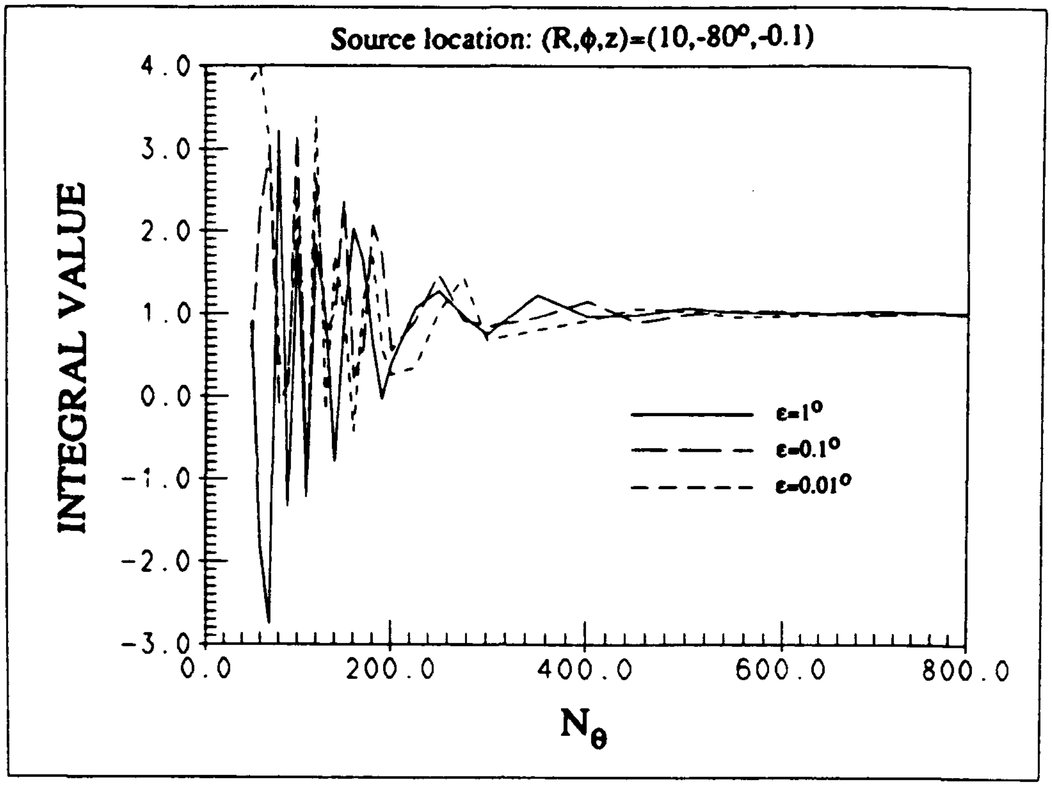

Figure 9. Convergence of integral (17) for (R,ϕ,z) = (10, -80°, -0.1)

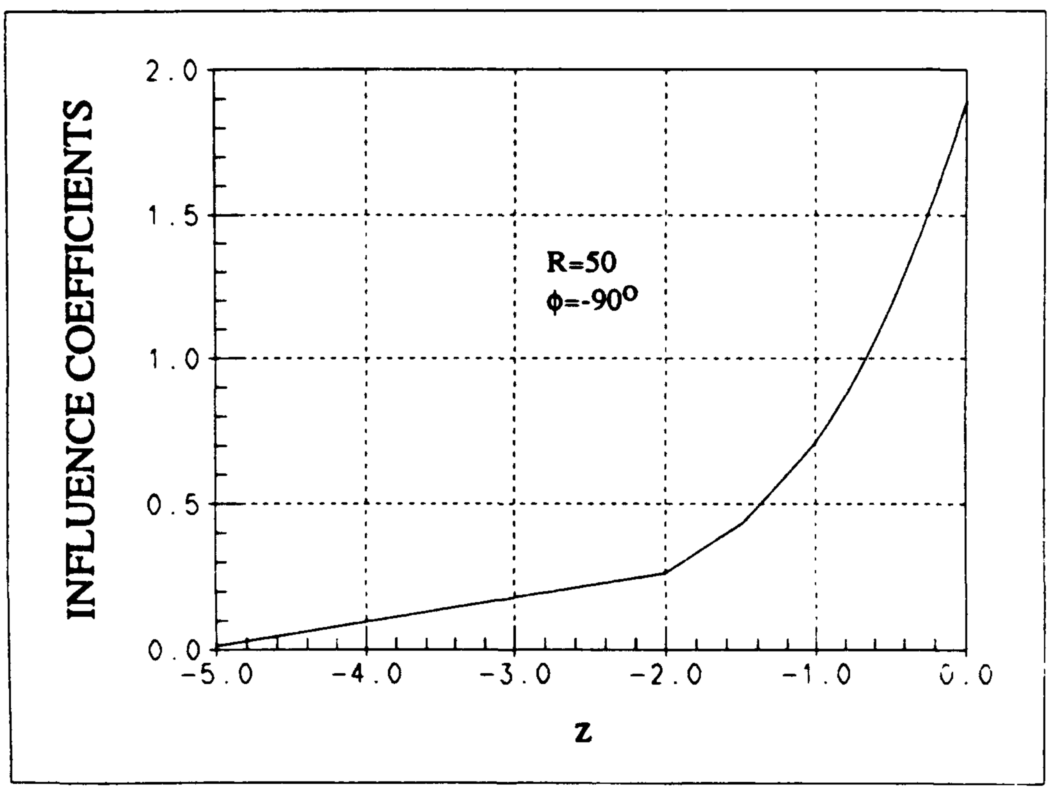

Figure 10. Influence coefficients vs. z for R = 50 , ϕ = -90°

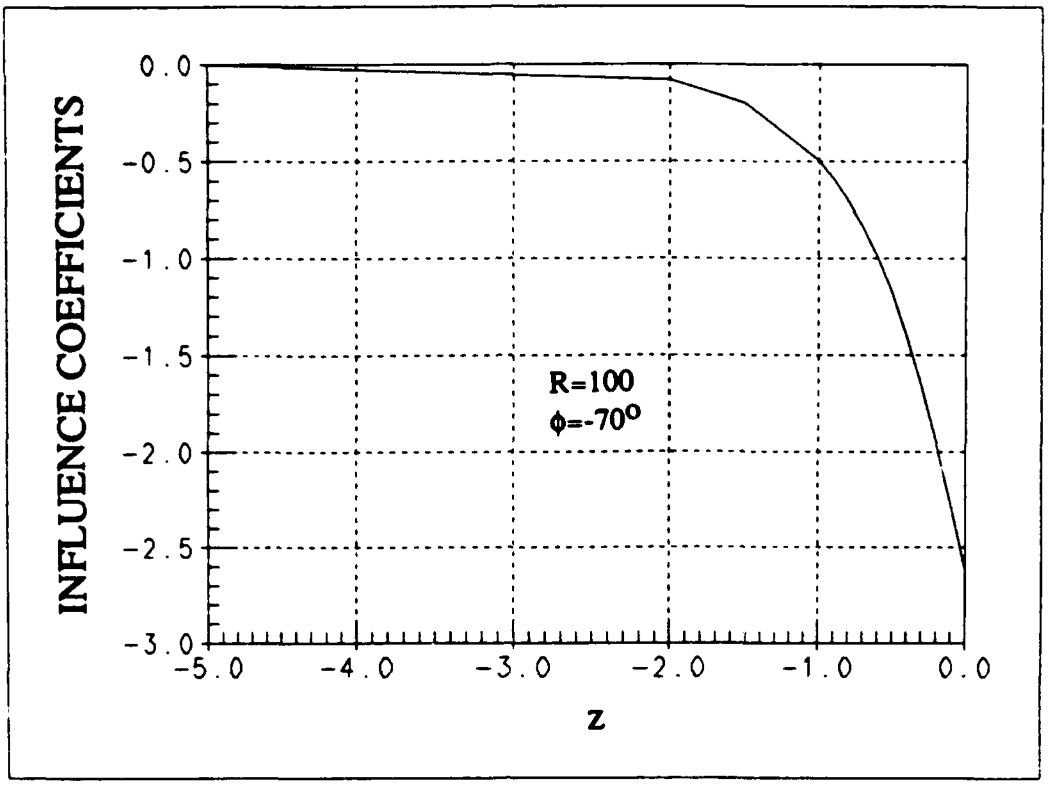

Figure 11. Influence coefficients vs. z for R = 100 , ϕ = -70°

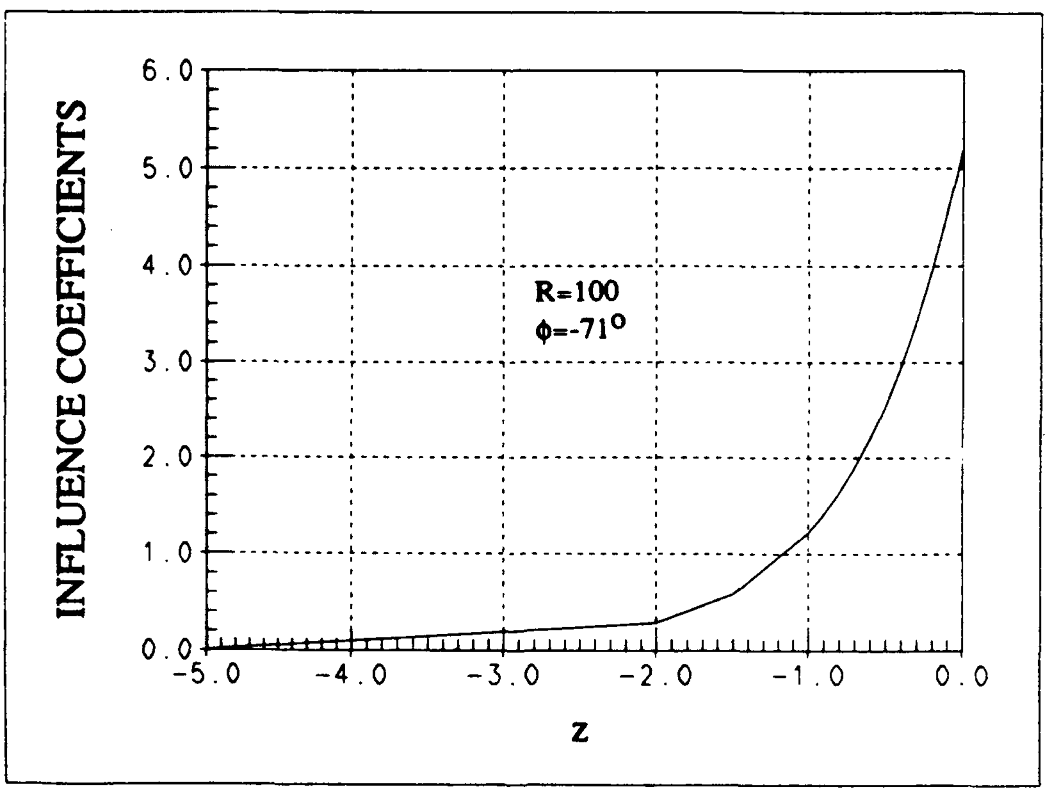

Figure 12. Influence coefficients vs. z for R = 100 , ϕ = -71°

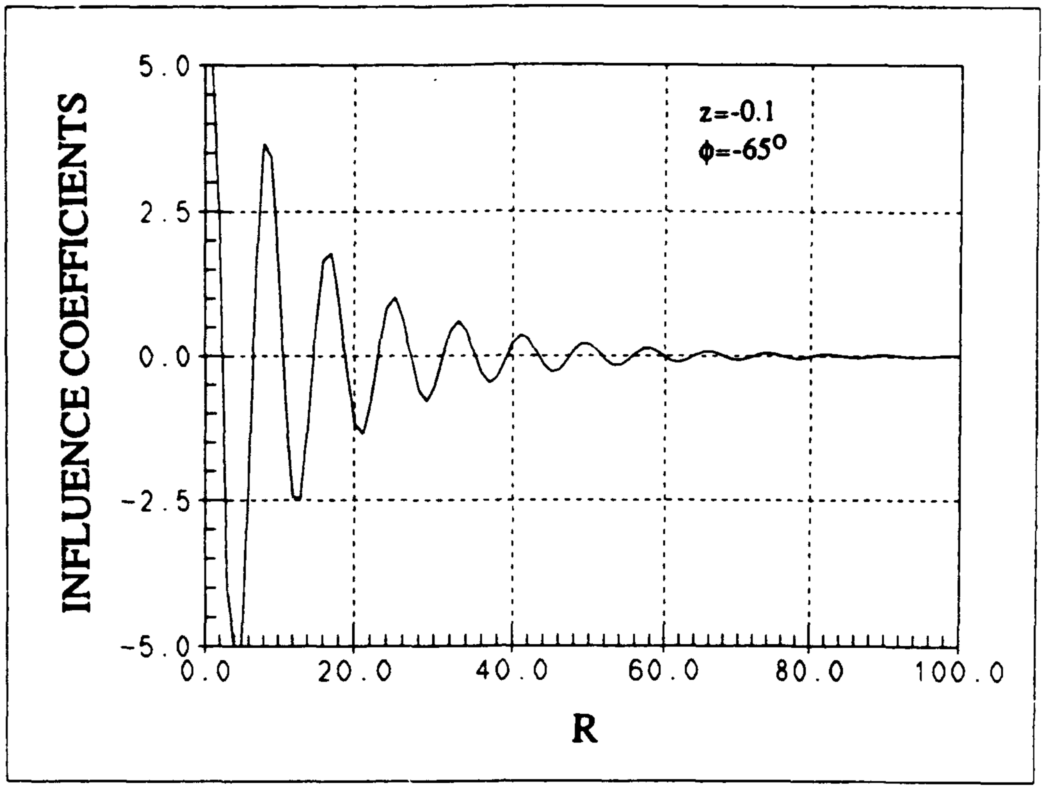

Figure 13. Influence coefficients vs. R for z = -0.1 , ϕ = -65°

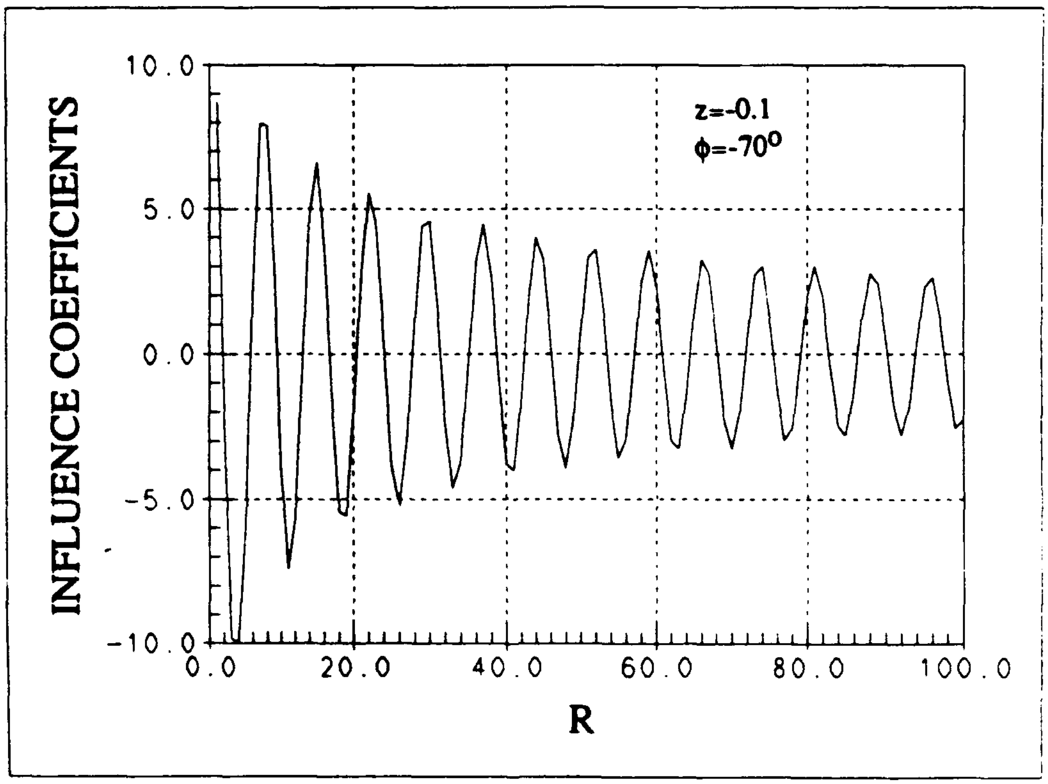

Figure 14. Influence coefficients vs. R for z = -0.1 , ϕ = -70°

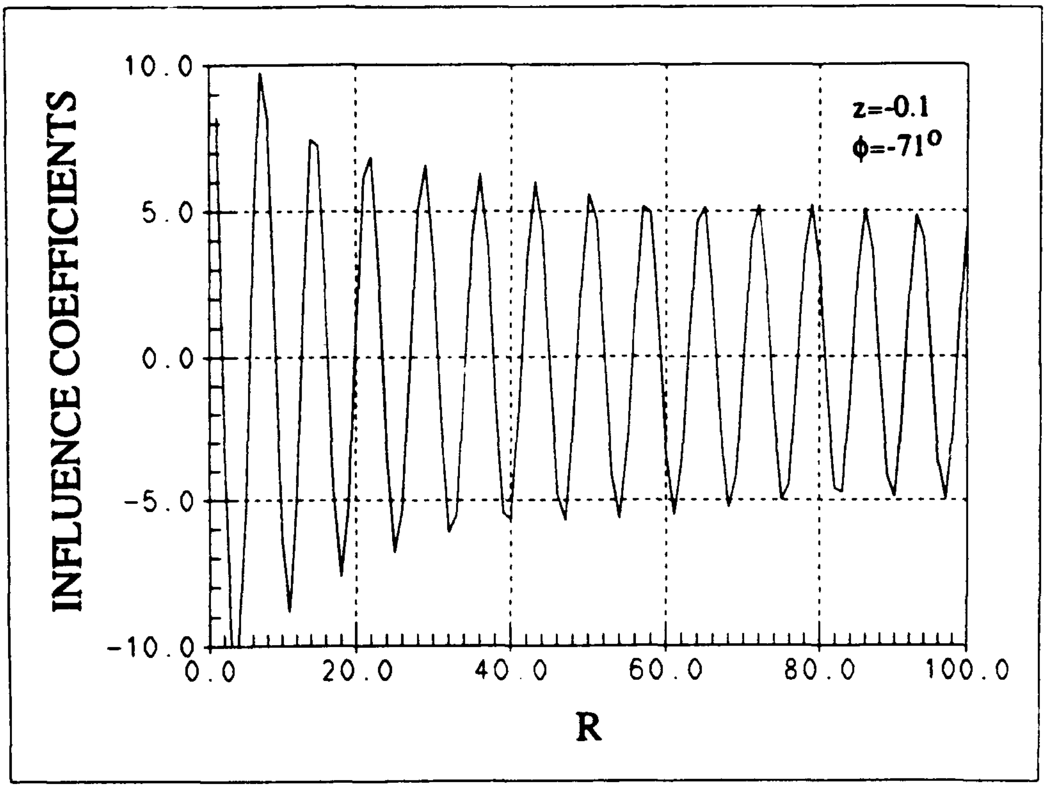

Figure 15. Influence coefficients vs. R for z = -0.1 , φ = -71°

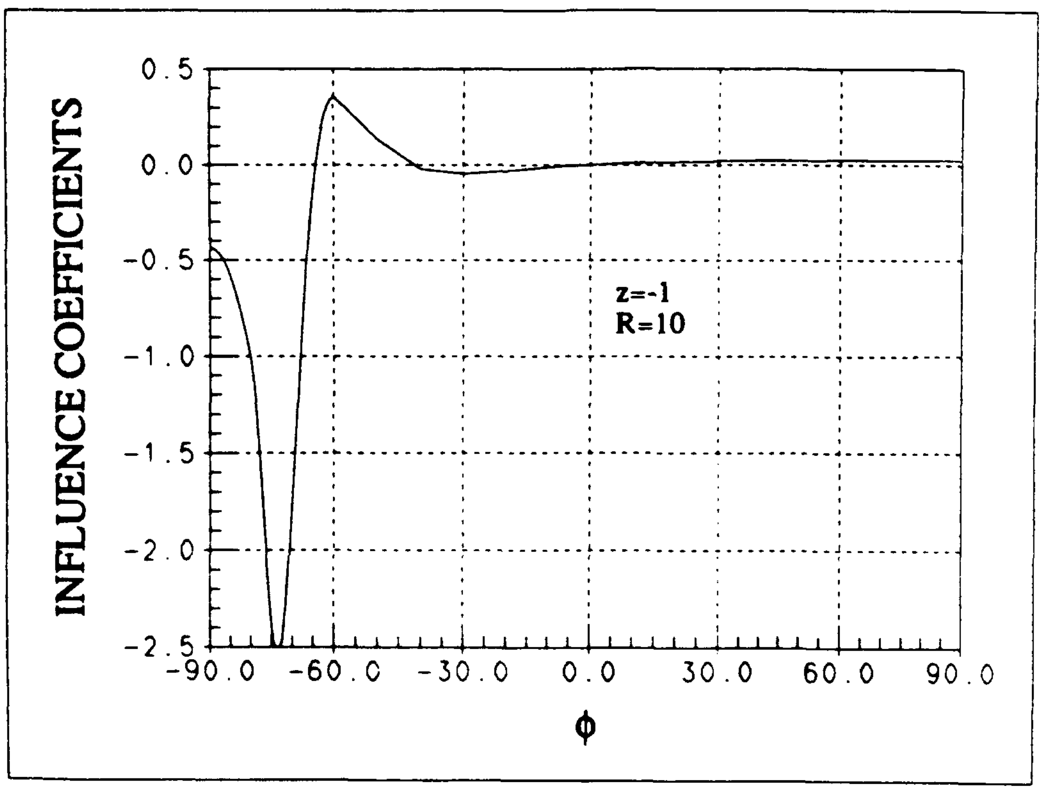

Figure 16. Influence coefficients vs. φ for z = -1 , R = 10

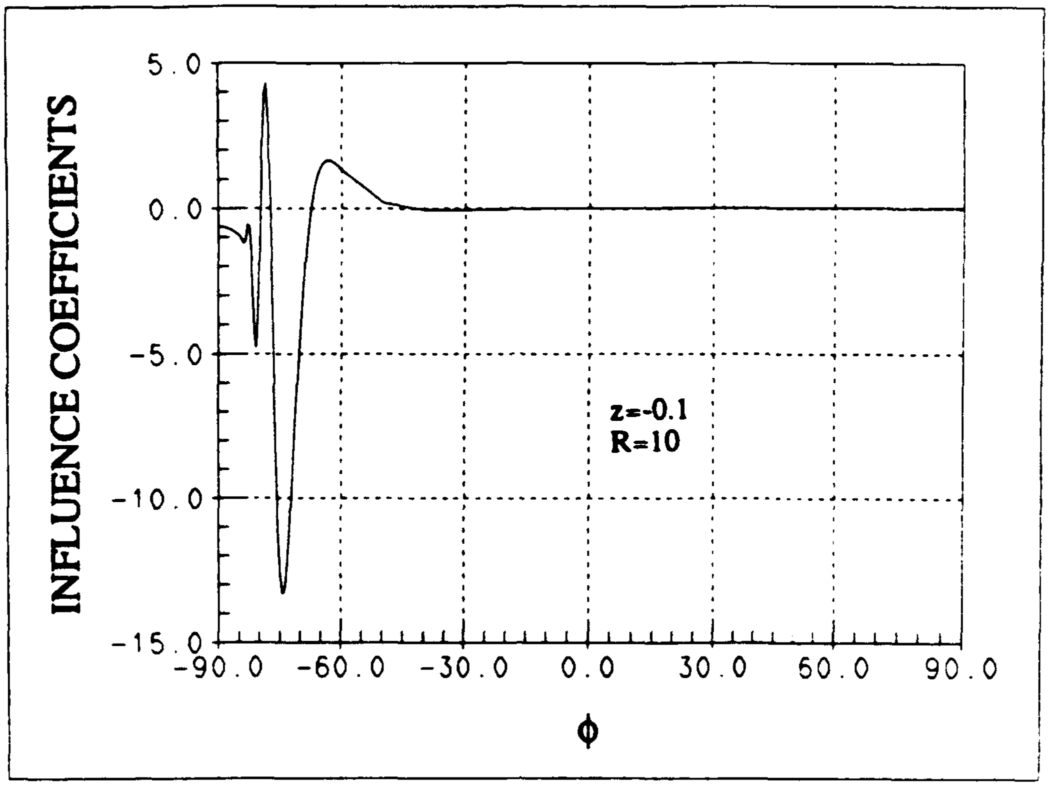

Figure 17. Influence coefficients vs. ϕ for z = -0.1 , R = 10

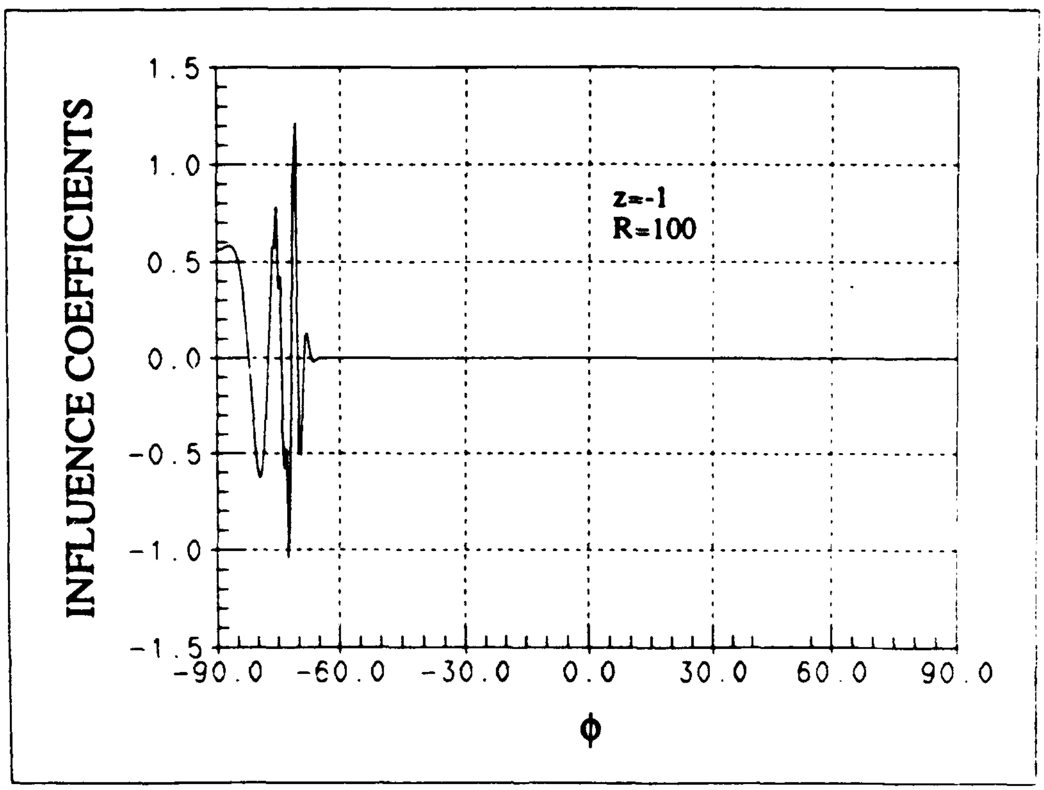

Figure 18. Influence coefficients vs. ϕ for z = -1 , R = 100

III. RESULTS

Kelvin wake calculations for the QUAPAW (see Table 1) at three Froude numbers are presented. Source strengths were computed using the Neumann-Kelvin solver DOCTORS.N-K. The original data file XYZFS was used to create quadrilateral panel data for the QUAPAW. This resulted in 192 panels per ship side; 24 in the longitudinal direction and 8 in the vertical. In the following pages contour plots and longitudinal wave cuts are included. Computations performed by WAVEAMP required less than 2 seconds CPU time per field point on an Apollo DN 3000 mini computer.

Table I

QUAPAW Model Dimensions

LWL = 16.250 ft	Wetted Surface = 64.883 ft^2
Beam = 3.208 ft	Scale Ratio = 12.00
Draft = 1.187 ft	

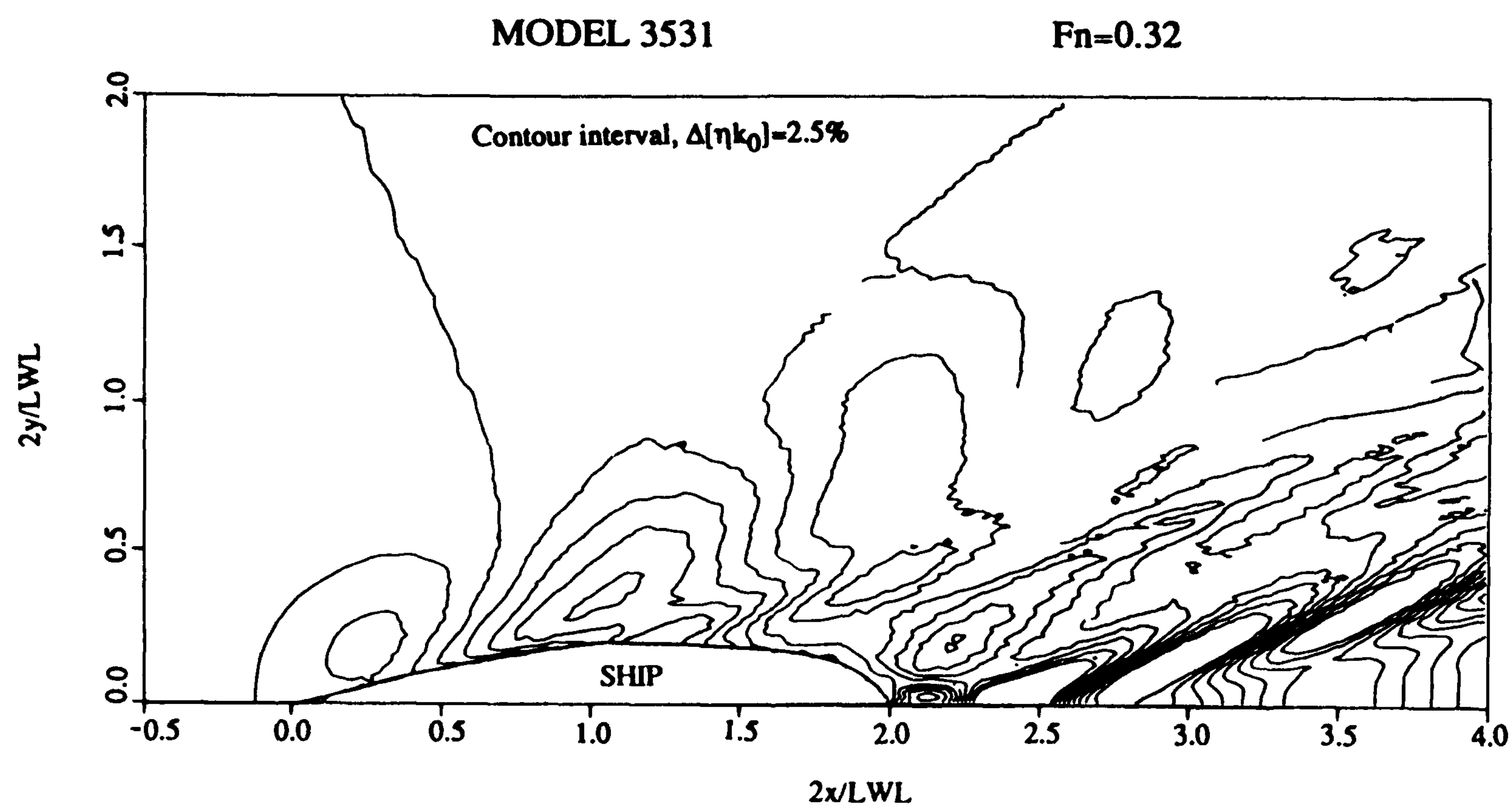

Figure 19. QUAPAW near-field wave pattern, Fn = 0.32

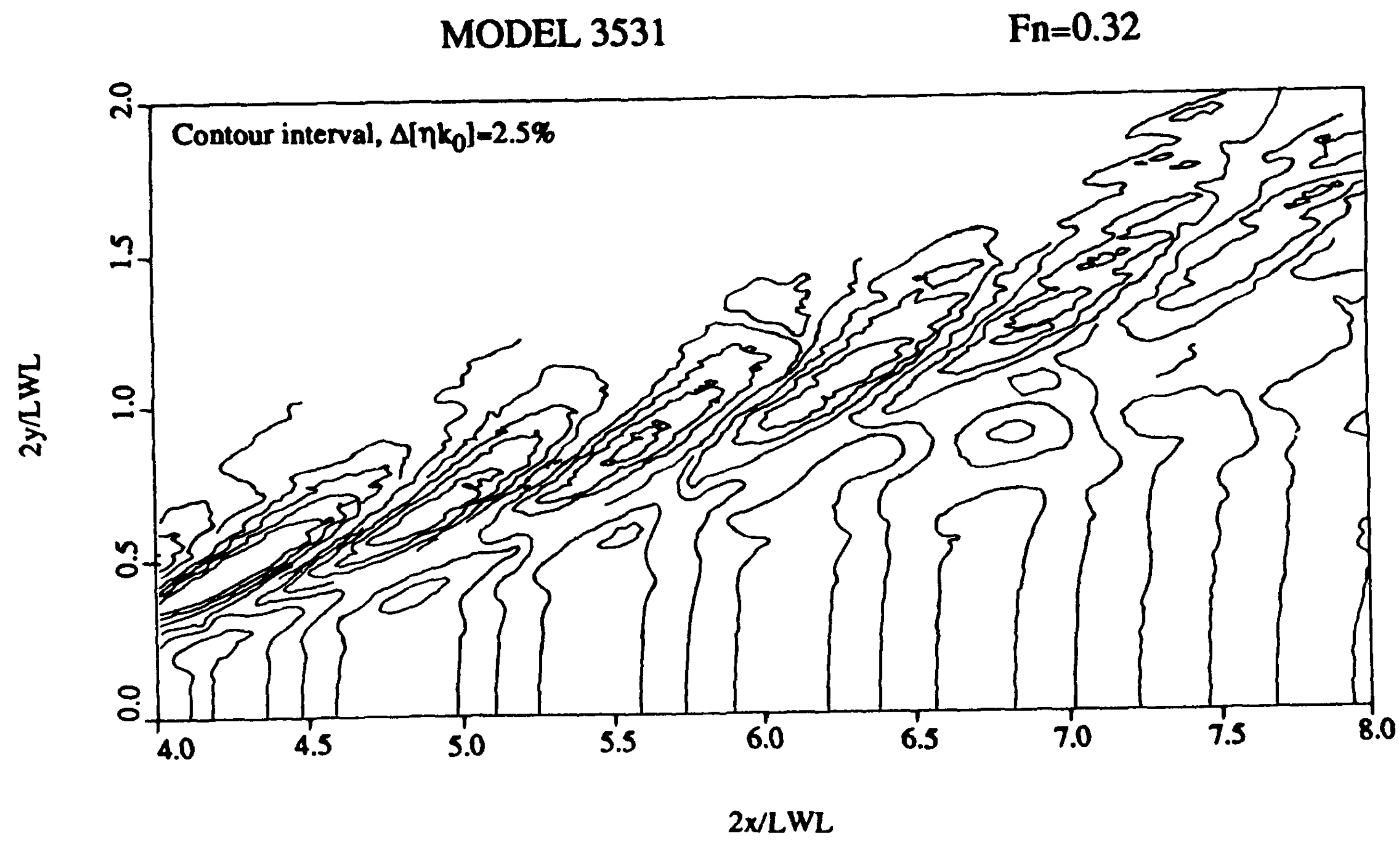

Figure 20. QUAPAW far-field wave pattern, Fn = 0.32

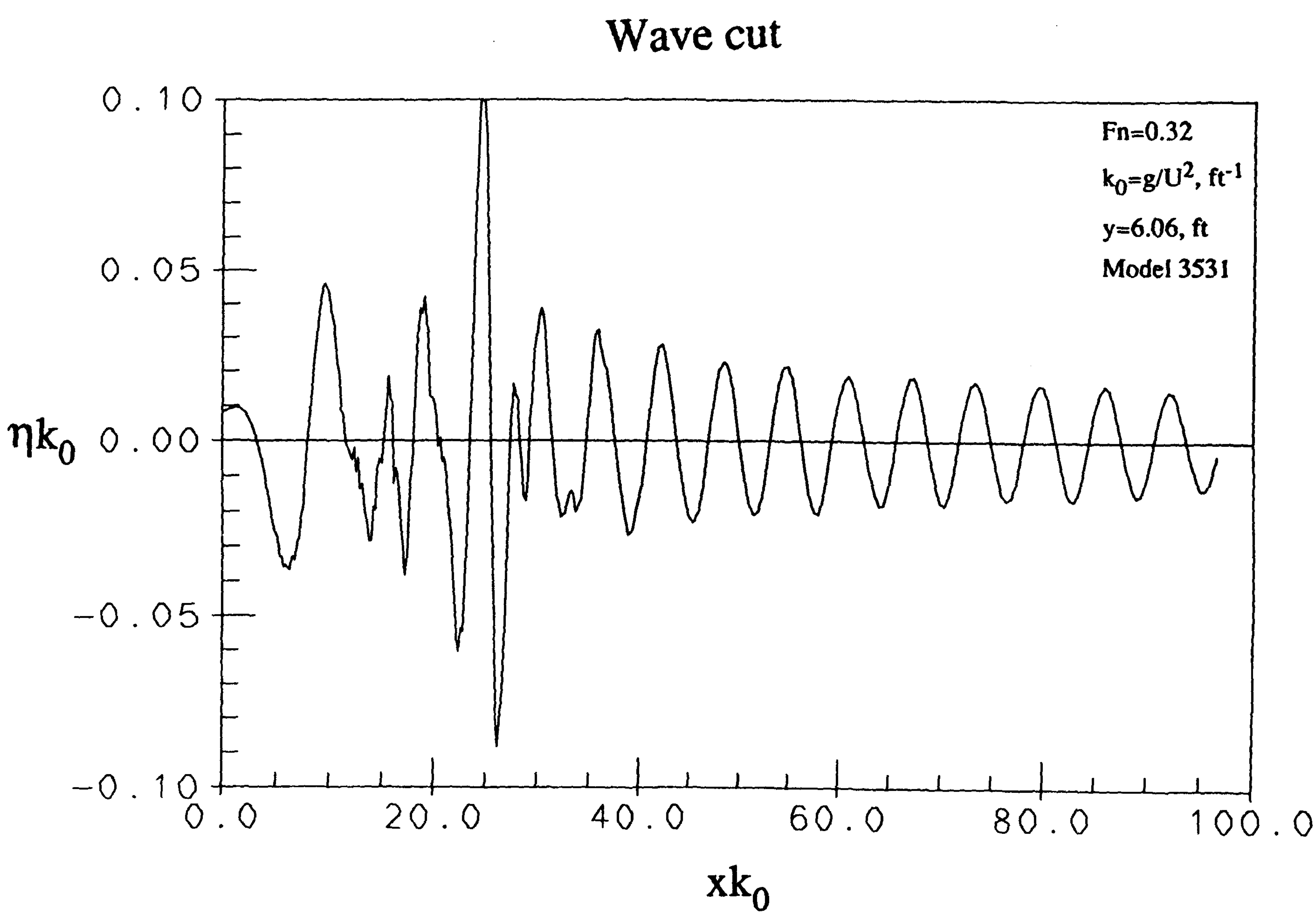

Figure 21. QUAPAW longitudinal wave-cut, Fn = 0.32

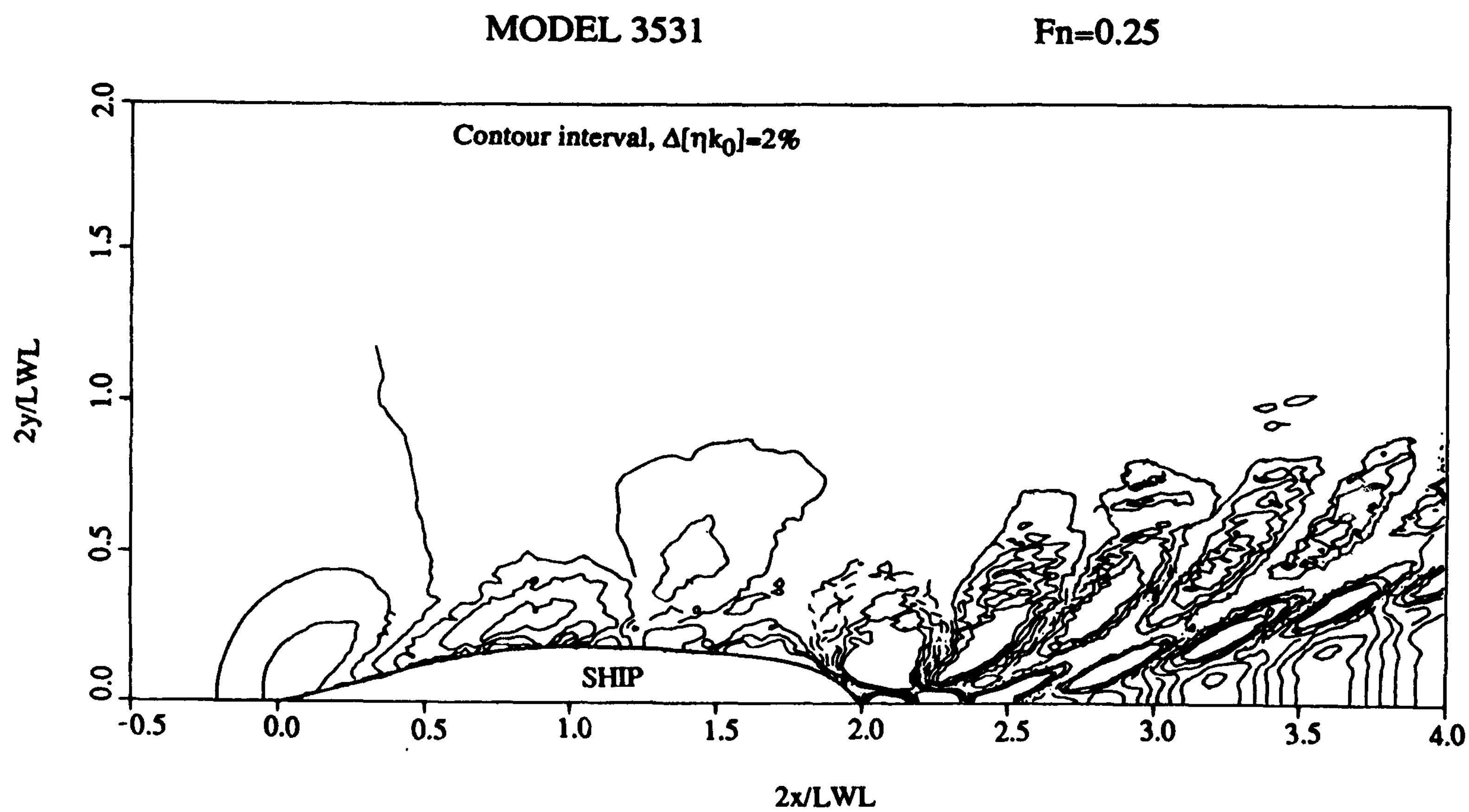

Figure 22. QUAPAW near-field wave pattern, Fn = 0.25

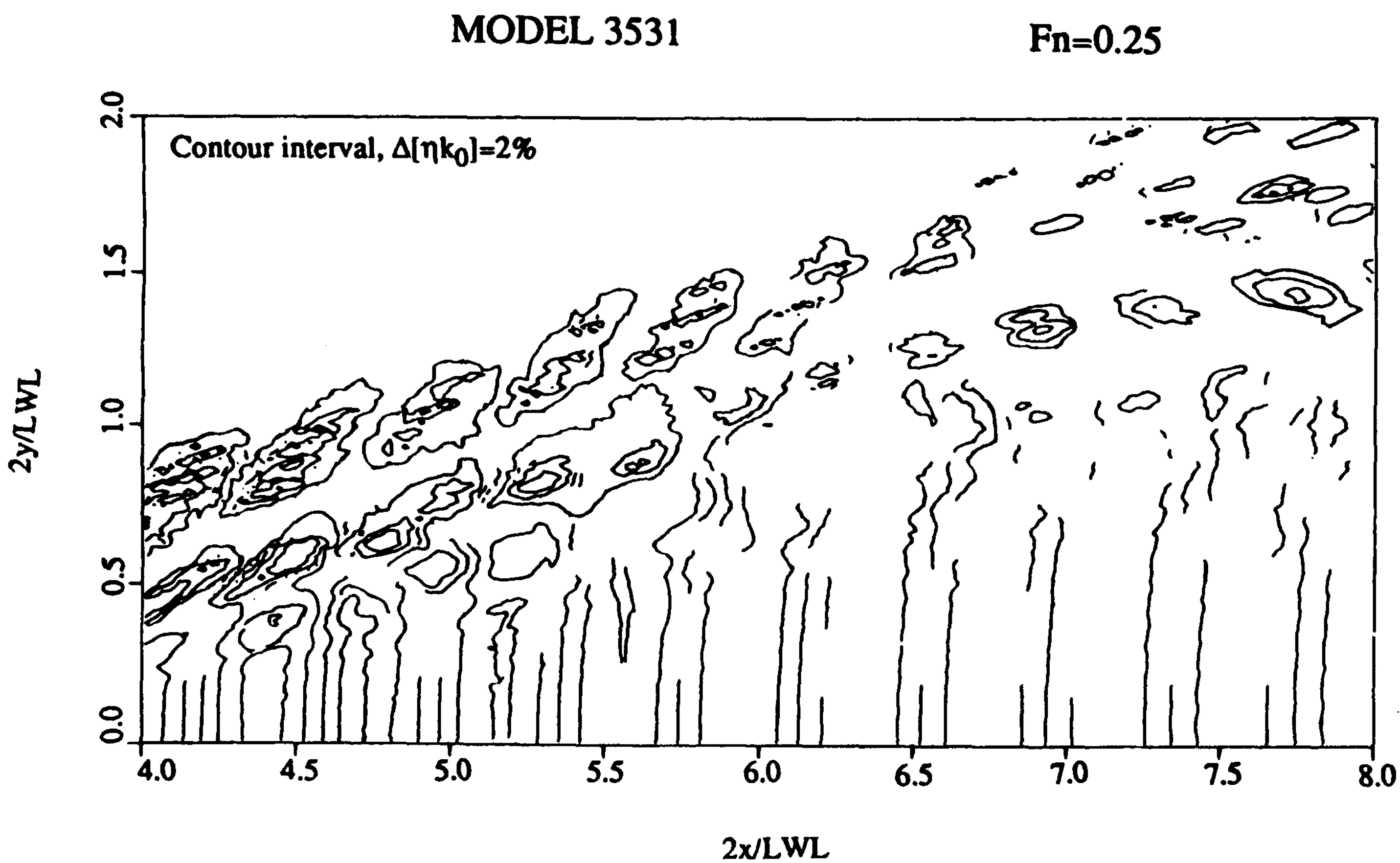

Figure 23. QUAPAW far-field wave pattern, Fn = 0.25

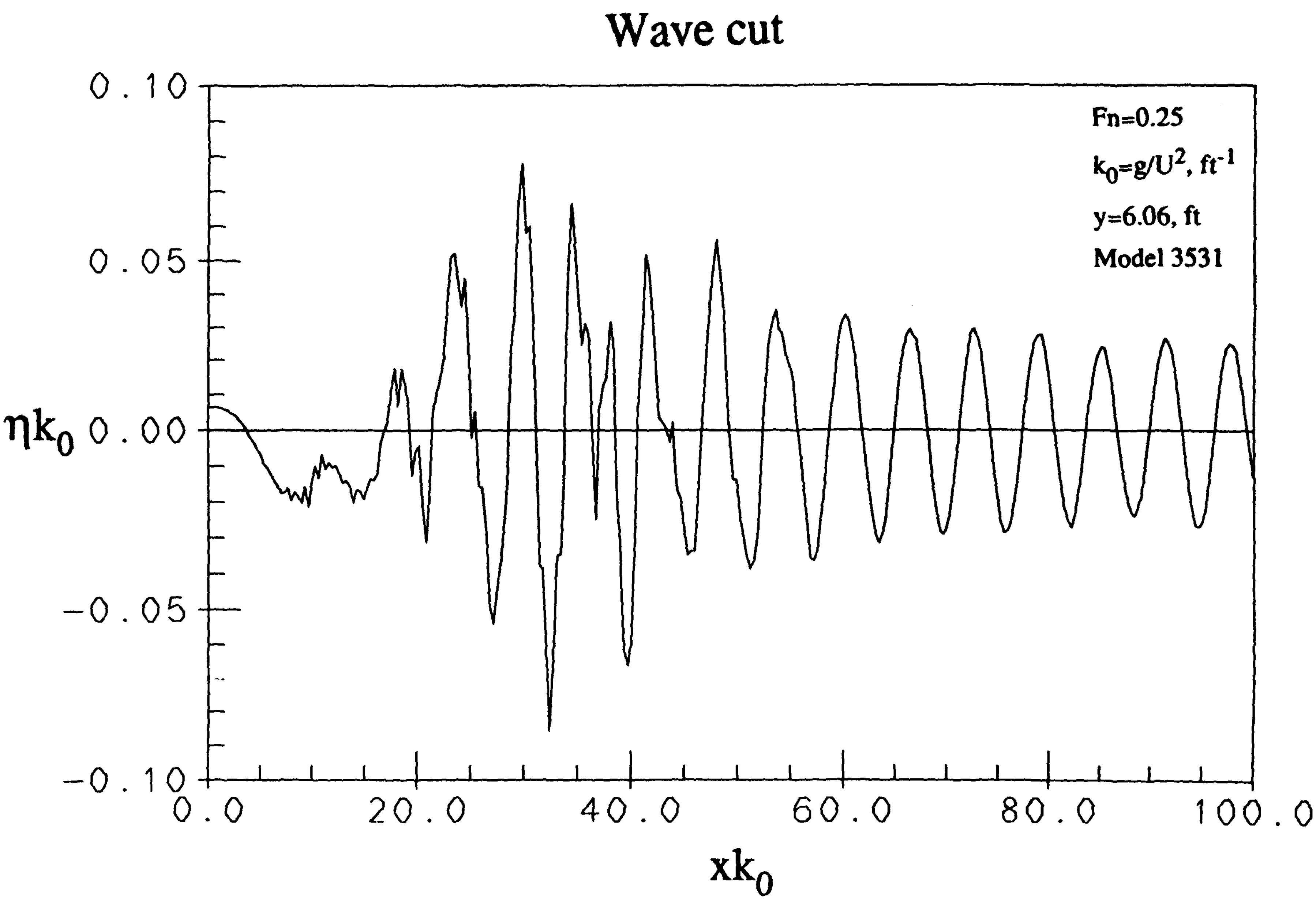

Figure 24. QUAPAW longitudinal wave-cut, Fn = 0.25

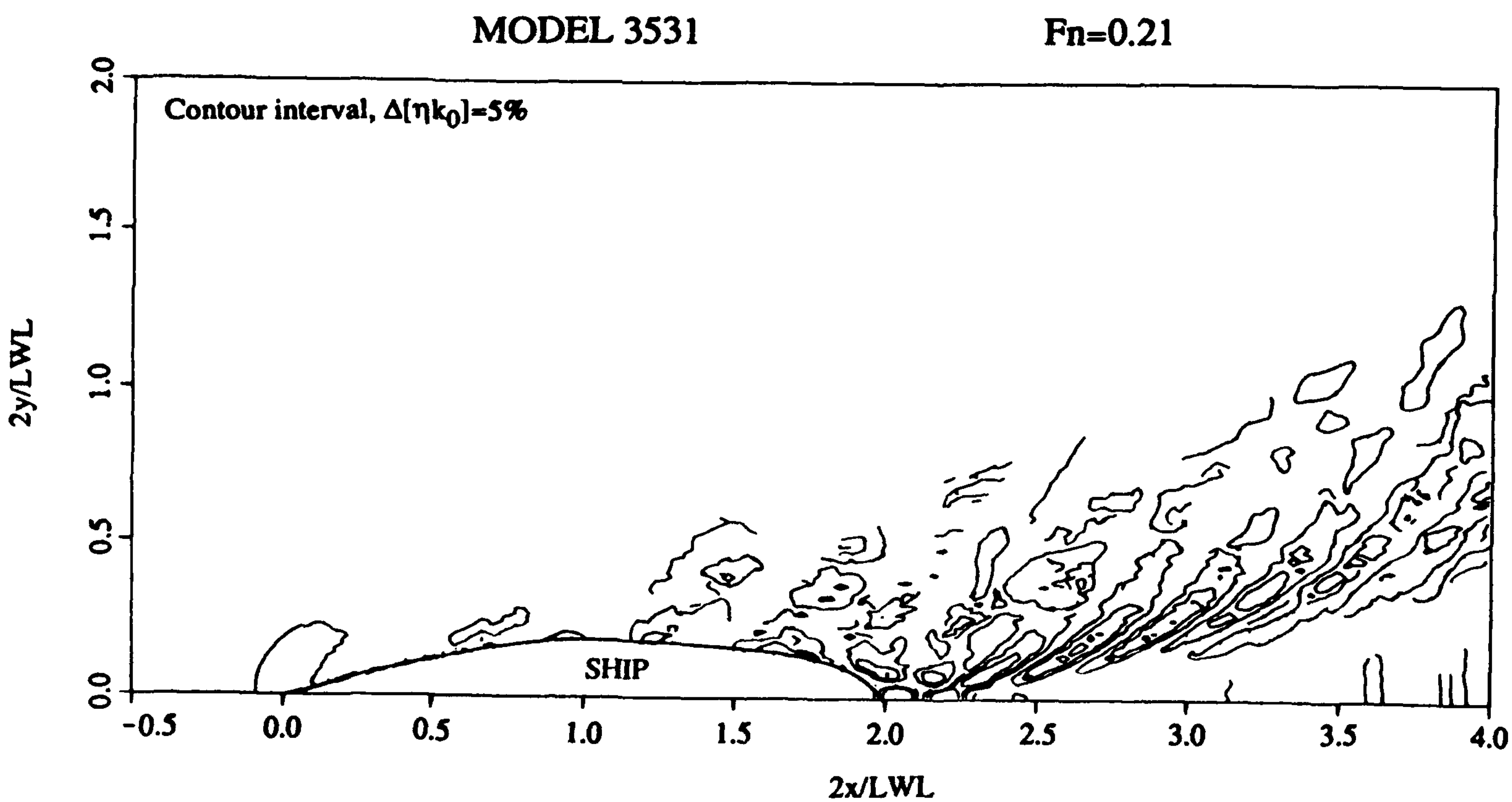

Figure 25. QUAPAW near-field wave pattern, Fn = 0.21

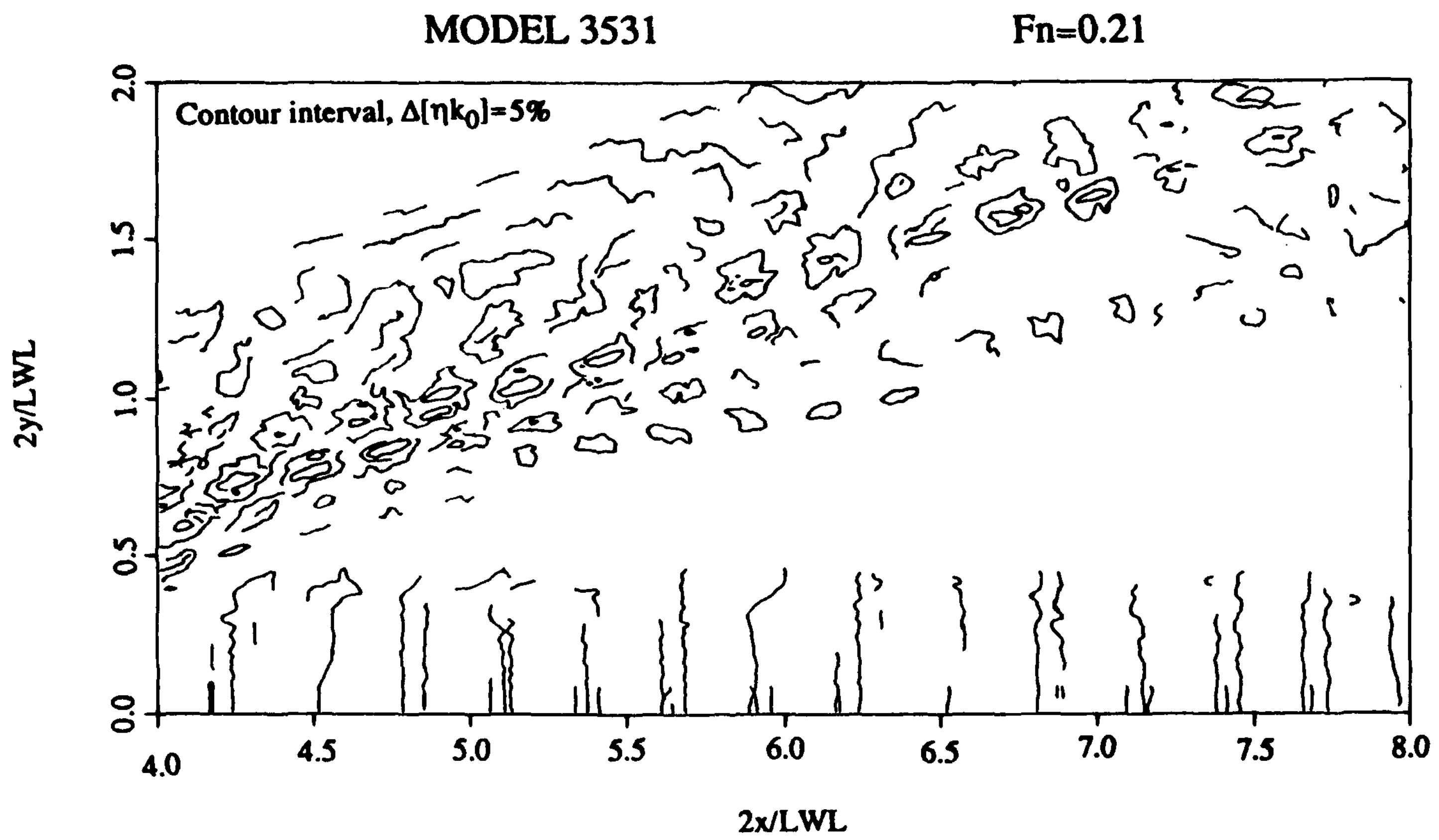

Figure 26. QUAPAW far-field wave pattern, Fn = 0.21

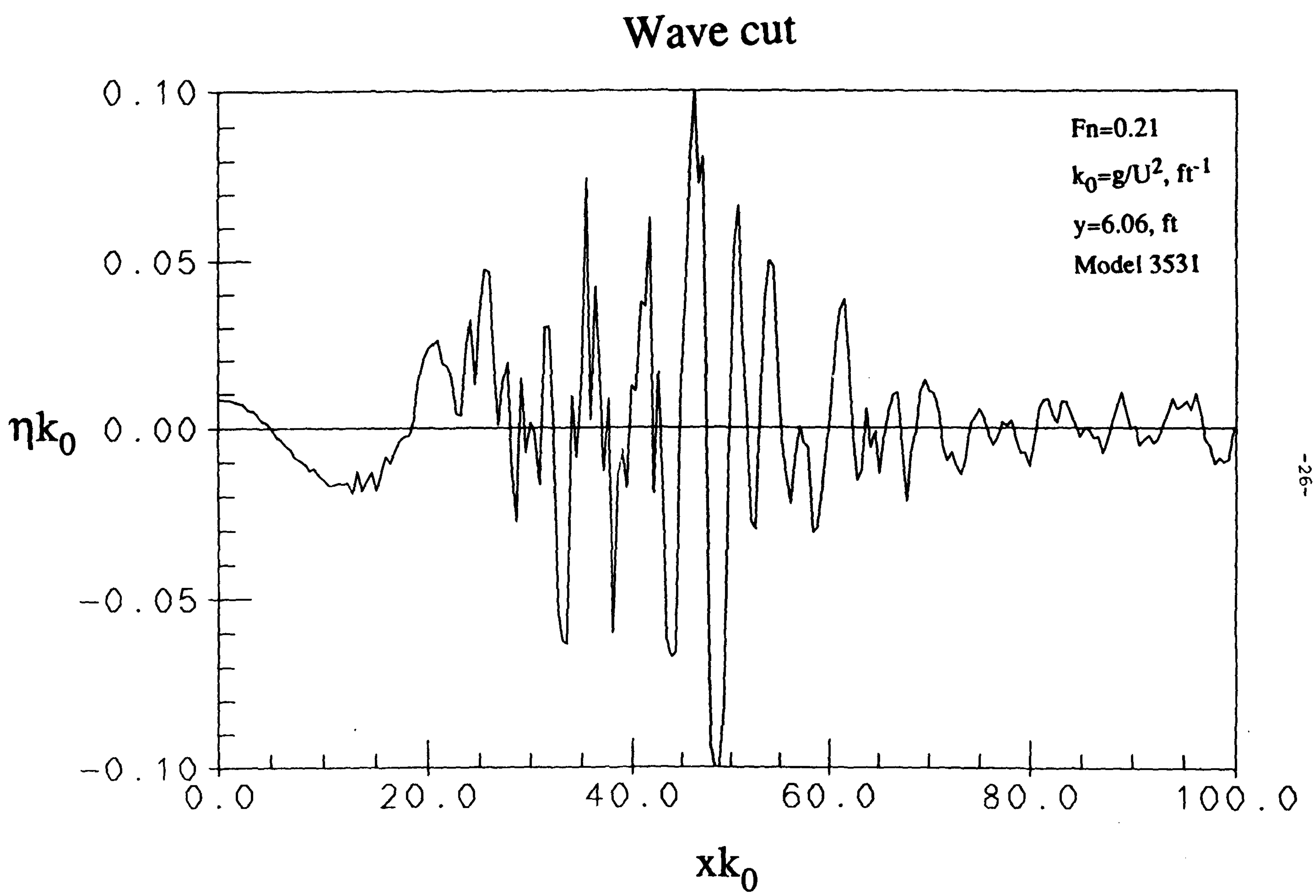

Figure 27. QUAPAW longitudinal wave-cut, Fn = 0.21

IV. PROGRAM DOCUMENTATION

IV.1 Description of Input

WAVEAMP needs three data files to run. File #1 contains the influence coefficients and is read through unit 1. File #2 contains wave resistance data, hull and panel data, and source strengths. It is read in on unit 5. This file is produced by running the Neumann-Kelvin solver DOCTORS.N-K. The user needs to prepare one data file, File #3, to be read on unit 4. This file contains the coordinates of the field points and is read through free format statements. The first line contains the total number of field points. There is no limitation on the maximum number of points. Each subsequent line contains the (x,y,z) coordinates of the particular field point (in feet) in that order. The following sign convention applies: x is zero at the ship's mid-section, positive upstream, and negative downstream; y is positive starboard; and z is zero.

IV.2 Description of Output

In addition to on-screen information output, WAVEAMP produces one results file written on unit 6. This file is identical to input File #3 except that the third column z is substituted by $\zeta(x,y)$, the wave elevation (in feet) at the particular field point. The wave elevation is positive for a wave crest and negative for a wave trough.

IV.3 Run Commands

The program is written in standard Fortran-77 for an Apollo mini-computer and can be easily transferred to any Fortran supporting machine. Execution begins (for the Apollo machine) by simply typing the name of the executable code and it prompts the user for the data and results files names.

IV.4 Error Messages

The only case where error messages may appear when running WAVEAMP is when the distance between source and field point is such that (R,ϕ,z) are out of their tabulated values range, as described in Section II. In such a case, execution is terminated and a self-explanatory error message is printed at the end of the results file.

CONCLUSIONS

The procedure of tabulating the values of the influence coefficients adopted by program WAVEAMP is an efficient and inexpensive alternative to time consuming but more accurate computations of the waves generated by a body moving in an inviscid, calm fluid. The achieved computational efficiency outweighs errors introduced by the interpolation scheme and simple monopole method. Finally, computational accuracy may be improved by using a finer grid of tabulated values and more accurate interpolation schemes.

REFERENCES

[1] Doctors, L.J. and Beck, R.F. (1987), Numerical aspects of the Newmann-Kelvin problem, Journal of Ship Research 31(1), 1-13.

[2] Doctors, L.J. and Beck, R.F. (1987), Convergence properties of the Newmann-Kelvin problem for a submerged body, Journal of Ship Research, 31(4), 227-234.

APPENDIX: PROGRAM LISTING

The following pages provide a complete listing of program WAVEAMP. The comments incorporated in the source code should be adequate to describe the flow of the code.

```
C     PROGRAM WAVEAMP
C
C     VERSION 3    : JUNE 1988
C
C     PROGRAMMER   : F.A. PAPOULIAS
C     DATE         : FEBRUARY 1988
C     LOCATION     : THE UNIVERSITY OF MICHIGAN
C
C     COMPILER     : STANDARD FORTRAN-77
C                    APOLLO COMPUTER
C
C     CALCULATION OF WAVE ELEVATIONS CREATED BY A THICK SHIP
C     TRAVELLING AT A CONSTANT SPEED BY THE NEUMANN-KELVIN METHOD
C     MONOPOLES APPROXIMATION
C
      DIMENSION VEE(200),SIGMA(200),XXO(3,4,200),
     1          XXQ(3,4,200),XXC(3,200),TRANS(9,200),ANN(3,200),
     2          D(4,200),TMAX(200),XXE(3,4,200),AA(8,200),AM(4,200),
     3          XXF(3,200),ICNT(4),WVCT(3),XXP(3,4,200)
C
      COMMON/DATA1/X1A(14),X2A(100),X3A(76)
      COMMON/DATA2/YA(14,100,76)
C
      DATA PI/ 3.1415 92653 5898/
C
      OPEN (UNIT=1,FILE='*INFLUENCE COEFF. FILE=')
      OPEN (UNIT=4,FILE='*FIELD POINTS FILE=')
      OPEN (UNIT=5,FILE='*WAVE RES. DATA FILE=')
      OPEN (UNIT=6,FILE='*RESULTS FILE=')
C
      WRITE (*,1002)
C
C     READ WAVE RESISTANCE DATA FILE
C
      READ (5,3001) NP,NX,NZ,NV,NH,IFREE,IPANEL,ISP,ISL
      READ (5,3004) TOLL
      READ (5,3002) (SIGMA(IP),IP=1,NP)
      READ (5,3002) (((XXO(I3,I4,IP),IP=1,NP),I4=1,4),I3=1,3)
      READ (5,3002) (((XXQ(I3,I4,IP),IP=1,NP),I4=1,4),I3=1,3)
      READ (5,3002) ((XXC(I3,IP),IP=1,NP),I3=1,3)
      READ (5,3002) ((TRANS(I9,IP),IP=1,NP),I9=1,9)
      READ (5,3002) ((ANN(I3,IP),IP=1,NP),I3=1,3)
      READ (5,3002) ((D(I4,IP),IP=1,NP),I4=1,4)
      READ (5,3002) (TMAX(IP),IP=1,NP)
      READ (5,3002) ((AA(I8,IP),IP=1,NP),I8=1,8)
      READ (5,3002) ((AM(I4,IP),IP=1,NP),I4=1,4)
      READ (5,3002) ((XXF(I3,IP),IP=1,NP),I3=1,3)
      READ (5,3003) RATIO1,RATIO2,G,V
      READ (5,3002) (((XXE(I3,I4,IP),IP=1,NP),I4=1,4),I3=1,3)
      READ (5,3002) (((XXP(I3,I4,IP),IP=1,NP),I4=1,4),I3=1,3)
C
      WRITE (*,1003)
C
C     READ INFLUENCE COEFFICIENTS DATA FILE
C
      DO 1 I1=1,14
        DO 2 I2=1,100
          READ (1,301) (YA(I1,I2,I76),I76=1,76)
    2   CONTINUE
    1 CONTINUE
C
C     LOAD VECTOR OF DEPTH COORDINATE Z
C
      DO 3 I=1,14
        IF (I.EQ.1) Z=-5.0
        IF (I.EQ.2) Z=-2.0
```

```
         IF (I.EQ.3) Z=-1.5
         IF (I.GT.3) Z=-1.0+(I-4)*0.1
         X1A(I)=Z
    3 CONTINUE
C
C     LOAD VECTOR OF RADIAL COORDINATE R
C
      DO 4 I=1,100
        R=I
        X2A(I)=I
    4 CONTINUE
C
C     LOAD VECTOR OF ANGULAR COORDINATE PHI
C
      DO 5 I=1,76
        PHI=90.0-10.0*(I-1)
        IF (I.GE.16) PHI=-60.0-0.5*(I-16)
        X3A(I)=PHI
    5 CONTINUE
C
      READ (4,*) NFP
C
C     LOOP OVER FIELD POINTS
C
      DO 800 KP=1,NFP
        WRITE (*,1001) KP,NFP
        READ (4,*) (WVCT(I),I=1,3)
C
C       INITIALISE VEE VECTOR
C
        DO 300 I=1,NP
          VEE(I)=0.0
  300   CONTINUE
C
C       CONSIDER BOTH SIDES OF THE SYMMETRY PLANE OF THE SHIP
C
        DO 450 IY=1,2
          IF (IY.EQ.2) WVCT(2)=-WVCT(2)
C
C         THE LINE-INTEGRAL CONTRIBUTION OF THE WAVE TERM
C
          FACLIN=V**2/(4.0*PI*G)
C
          CALL LINELV (NP,NX,4,WVCT,XXQ,ANN,G,V,FACLIN,VEE,IY)
C
C         THE SURFACE-INTEGRAL CONTRIBUTION OF THE WAVE TERM
C
          FACPAN=-1.0/(4.0*PI)
          CALL PANELV (NP,XXF,WVCT,AA,G,V,FACPAN,VEE)
C
          IF (IY.EQ.2) WVCT(2)=-WVCT(2)
  450   CONTINUE
C
C       COMPUTE WAVE ELEVATION
C
        DO 780 JP=1,NP
          WVCT(3)=(VEE(JP)*SIGMA(JP))*(V/G)+WVCT(3)
  780   CONTINUE
C
C       PRINT OUT RESULTS
C
        WRITE (6,2000) WVCT(1),WVCT(2),WVCT(3)
  800 CONTINUE
C
      STOP
C
C     FORMAT STATEMENTS
C
  301 FORMAT (5E15.5)
 1001 FORMAT (' FIELD POINT',I6,' OUT OF',I6,' TOTAL')
```

```
 1002 FORMAT (' READING WAVE RESISTANCE DATA FILE .....')
 1003 FORMAT (' READING INFLUENCE COEFFICIENTS .....')
 2000 FORMAT (3E15.5)
 3001 FORMAT (9I5)
 3002 FORMAT (5E20.10)
 3003 FORMAT (4E20.10)
 3004 FORMAT (E20.10)
C
      END
C
```

```
      SUBROUTINE PANELV (NP,XXC,WVCT,AAA,G,V,FACPAN,VEE)
C
C     THE WAVE PART OF THE VELOCITY INDUCED AT A FIELD POINT
C     BY A UNIFORM SOURCE PANEL
C
      DIMENSION  XXC(3,NP),WVCT(3),AAA(8,NP),VEE(NP)
C
      COMMON/DATA1/X1A(14),X2A(100),X3A(76)
      COMMON/DATA2/YA(14,100,76)
C
      DATA PI/ 3.1415 92653 5898/
C
      AK0=G/V**2
C
C     STEP THROUGH THE SOURCE PANELS
C
      DO 300 JP=1,NP
        DX=WVCT(1)-XXC(1,JP)
        DY=WVCT(2)-XXC(2,JP)
        DZ=WVCT(3)+XXC(3,JP)
        DY=ABS(DY)
        DZ=-ABS(DZ)
        DX=DX*AK0
        DY=DY*AK0
        DZ=DZ*AK0
        IF (DZ.LT.(-5.0))  GO TO 8001
        IF (DZ.GE.0.0)     GO TO 8002
        R=SQRT(DX*DX+DY*DY)
        IF (R.LT.1.0)    GO TO 8003
        IF (R.GE.100.0)  GO TO 8004
        PHI=ASIN(ABS(DX)/R)
        PHI=PHI*180.0/PI
        IF (DX.LT.0.0) PHI=-PHI
        CALL LOCATE(X1A, 14, DZ,I)
        CALL LOCATE(X2A,100,  R,J)
        CALL LOCATE(X3A, 76,PHI,K)
        CALL TRLINT(14,100,76,I,J,K,YA,X1A,X2A,X3A,DZ,R,PHI,SUMX)
        FAC=-2.0*AK0**2*AAA(1,JP)/PI
        VXX=FAC*SUMX
        VEE(JP)=VEE(JP)+FACPAN*VXX
300   CONTINUE
C
      RETURN
C
C     AB-NORMAL OUTPUT
C
 8001 WRITE (6,7001) JP
      STOP 9001
 8002 WRITE (6,7002) JP
      STOP 9002
 8003 WRITE (6,7003) JP
      STOP 9003
 8004 WRITE (6,7004) JP
      STOP 9004
 7001 FORMAT (/,' Z IS LESS THAN -5 FOR PANEL NO.',I5)
 7002 FORMAT (/,' Z IS GREATER THAN 0 FOR PANEL NO.',I5)
 7003 FORMAT (/,' R IS LESS THAN 1 FOR PANEL NO.',I5)
 7004 FORMAT (/,' R IS GREATER THAN 100 FOR PANEL NO.',I5)
C
      END
C
```

```
      SUBROUTINE LINELV (NP,NX,NV,WVCT,XXQ,ANN,G,V,FACLIN,VEE,IFAP)
C
C     THE WAVE PART OF THE VELOCITY INDUCED AT A FIELD POINT
C     BY A UNIFORM SOURCE LINE
C
      DIMENSION XXQ(3,NV,NP),WVCT(3),ANN(3,NP),VEE(NP)
C
      COMMON/DATA1/X1A(14),X2A(100),X3A(76)
      COMMON/DATA2/YA(14,100,76)
C
      DATA PI/ 3.1415 92653 5898/
C
      AK0=G/V**2
      TOL=1.0E-05
      NX1=NX-1
C
C     STEP THROUGH THE SOURCE LINES
C
      DO 300 JX=1,NX1
        DX=WVCT(1)-0.5*(XXQ(1,4,JX)+XXQ(1,1,JX))
        DY=WVCT(2)-0.5*(XXQ(2,4,JX)+XXQ(2,1,JX))
        DZ=WVCT(3)
        DY=ABS(DY)
        DZ=-ABS(DZ)
        DX=DX*AK0
        DY=DY*AK0
        DZ=DZ*AK0
        DDY=XXQ(2,1,JX)-XXQ(2,4,JX)
        IF (ABS(DDY).LT.TOL) GO TO 300
        IF (DZ.LT.(-5.0)) GO TO 8001
        IF (DZ.GE.0.0) DZ=0.001
        R=SQRT(DX*DX+DY*DY)
        IF (R.LT.1.)    GO TO 8003
        IF (R.GE.100.0) GO TO 8004
        PHI=ASIN(ABS(DX)/R)
        PHI=PHI*180.0/PI
        IF (DX.LT.0.0) PHI=-PHI
        CALL LOCATE(X1A, 14, DZ,I)
        CALL LOCATE(X2A,100,  R,J)
        CALL LOCATE(X3A, 76,PHI,K)
        CALL TRLINT(14,100,76,I,J,K,YA,X1A,X2A,X3A,DZ,R,PHI,SUMX)
        FAC=2.0*DDY*AK0**2*ANN(1,JX)/PI
        VX=FAC*SUMX
        IF (IFAP.EQ.2) VX=-VX
        VEE(JX)=VEE(JX)+FACLIN*VX
300   CONTINUE
C
      RETURN
C
C     AB-NORMAL OUTPUT
C
 8001 WRITE (6,7001) JP
      STOP 9001
 8003 WRITE (6,7003) JP
      STOP 9003
 8004 WRITE (6,7004) JP
      STOP 9004
 7001 FORMAT (/,' Z IS LESS THAN -5 FOR PANEL NO.',I5)
 7003 FORMAT (/,' R IS LESS THAN 1 FOR PANEL NO.',I5)
 7004 FORMAT (/,' R IS GREATER THAN 100 FOR PANEL NO.',I5)
C
      END
C
```

```
      SUBROUTINE LOCATE (XX,N,X,J)
C
C     GIVEN AN ARRAY XX OF LENGTH N, AND GIVEN A VALUE X, RETURNS
C     A VALUE J SUCH THAT X IS BETWEEN XX(J) AND XX(J+1). XX MUST
C     BE MONOTONIC, EITHER DECREASING OR INCREASING. J=0 OR J=N
C     IS RETURNED TO INDICATE THAT X IS OUT OF RANGE
C
      DIMENSION XX(N)
C
      JL=0
      JU=N+1
   10 IF (JU-JL.GT.1) THEN
        JM=(JU+JL)/2
        IF ((XX(N).GT.XX(1)).EQV.(X.GT.XX(JM))) THEN
          JL=JM
        ELSE
          JU=JM
        ENDIF
      GO TO 10
      ENDIF
      J=JL
      RETURN
      END
C
```

```
      SUBROUTINE TRLINT(L,M,N,I,J,K,YA,X1A,X2A,X3A,X1,X2,X3,Y)
C
C     LINEAR INTERPOLATION IN THREE DIMENSIONS.
C     GIVEN A THIRD ORDER TENSOR YA(L,M,N); ARRAYS X1A(L),
C     X2A(M), X3A(N); THREE NUMBERS X1,X2,X3 AND I,J,K SUCH
C     THAT X1 IS BETWEEN X1A(I) AND X1A(I+1), X2 IS BETWEEN
C     X2A(J) AND X2A(J+1), AND X3 IS BETWEEN X3A(K) AND
C     X3A(K+1); IT RETURNS Y(X1,X2,X3) BY TRILINEAR
C     INTERPOLATION IN THE TENSOR YA.
C
      DIMENSION YA(L,M,N),X1A(L),X2A(M),X3A(N)
C
      Y1=YA(I  ,J  ,K  )
      Y2=YA(I  ,J+1,K  )
      Y3=YA(I  ,J+1,K+1)
      Y4=YA(I  ,J  ,K+1)
      Y5=YA(I+1,J  ,K  )
      Y6=YA(I+1,J+1,K  )
      Y7=YA(I+1,J+1,K+1)
      Y8=YA(I+1,J  ,K+1)
C
      T=(X1-X1A(I))/(X1A(I+1)-X1A(I))
      U=(X2-X2A(J))/(X2A(J+1)-X2A(J))
      W=(X3-X3A(K))/(X3A(K+1)-X3A(K))
C
      T1=1.0-T
      U1=1.0-U
      W1=1.0-W
C
      Y=T1*U1*W1*Y1+T*U1*W1*Y2+T*U*W1*Y3+T1*U*W1*Y4+
     .  T1*U1*W *Y5+T*U1*W *Y6+T*U*W *Y7+T1*U*W *Y8
C
      RETURN
      END
```